Chapter 2.2 Range Symmetry

From *Explico Playground — Motion*, with changes after version 1.0.0, not yet released. Interactive version: https://danieltuzes.github.io/explico/playground/projectiles/range-symmetry/

The range formula in (2.1.1) predicts a symmetry that is useful when comparing trajectories. With the same Launch speed, a low launch and a high launch can have the same Range.

2.2.1 Complementary angles

Let the Launch angle be θ. Its complementary angle is $90^\circ - \theta$, because the two angles add to 90°. Doubling the complementary angle and using the supplementary-angle identity gives the same sine factor as the original angle:

$$\begin{aligned} \sin(2(90^\circ - \theta)) &= \sin(180^\circ - 2\theta) \\ &= \sin(2\theta) \end{aligned} \tag{2.2.1}$$

The other factors in the range formula do not change. Therefore complementary launch angles give the same range over level ground when air resistance is ignored.

2.2.2 Worked example: matching range

For a launch at 30°, the complementary angle is $90^\circ - 30^\circ = 60^\circ$. Their doubled angles are 60° and 120°, which have the same sine. Therefore launches at 30° and 60° travel the same horizontal distance when their launch speeds are equal.

2.2.3 Different paths, one range

Equal range does not mean equal motion. The steeper launch rises higher and stays in the air longer. The shallower launch has a larger horizontal velocity component, which compensates for its shorter time in the air.

2.2.4 Checking the pairs numerically

A short script computes the range of each launch angle and of its complement (the angle that adds up to 90° with it), for the same launch speed:

range_pairs.py

```
from math import radians, sin

def launch_range(v0, angle_deg, g=9.81):
    return v0**2 * sin(radians(2 * angle_deg)) / g

for angle in (15, 30, 45):
    print(angle, 90 - angle, launch_range(31.6, angle))
```

Its output, one row per pair, is in the table of range pairs. A common slip is to pair 30° with 150° 60°: the two angles add up to 90°, not to 180°. Figure 2.2.1 sums up why each pair shares its range.

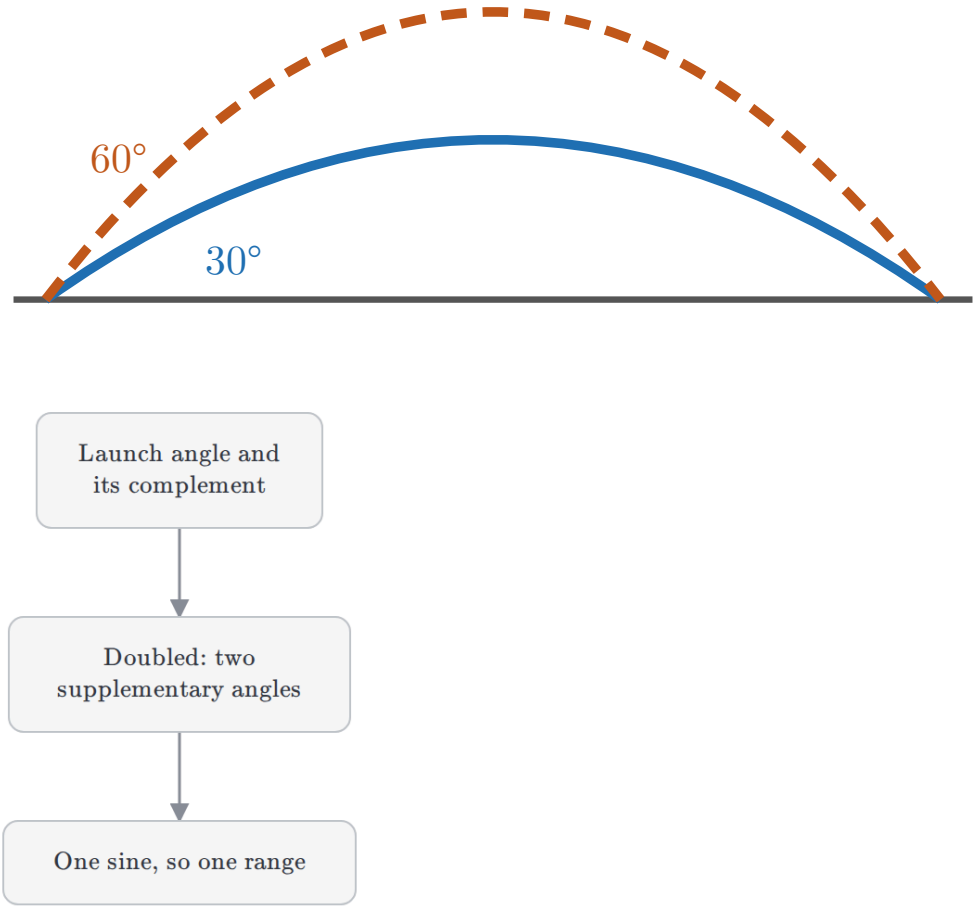


Figure 2.2.1 From an angle to its partner. Each launch angle and its complement double to supplementary angles, which share one sine and so one range.

Before the checks, make sure of both steps:

- ☒ the range formula over level ground
- ☐ the supplementary-angle identity for the sine

Knowledge check 2.2.1: Complementary launch angles

1. A projectile is launched at 35° over level ground with no air resistance. Which other launch angle gives the same range at the same launch speed?
   - (a) 25°
   - (b) 45°
   - (c) 55°

- (d) 65°

Answers: end of the chapter.

Notation

g **Gravitational acceleration**. The magnitude of gravitational acceleration, treated as constant near the ground. *Units:* meters per second squared.

θ **Launch angle**. The angle between a projectile's initial velocity and the horizontal ground at launch. *Units:* degrees above the horizontal.

v_0 **Launch speed**. The object's speed at the start of the scenario being analyzed — its initial velocity in a straight-line problem, or its launch speed the instant a projectile leaves the ground. *Units:* meters per second.

R **Range**. Total horizontal distance a projectile covers before it returns to its launch height. *Units:* meters.

$$R = \frac{{v_0}^2 \sin(2\theta)}{g}$$

t **Time**. Elapsed time since the object was launched or released, measured in seconds. *Units:* seconds after launch.

Answers to the knowledge checks

Knowledge check 2.2.1: Complementary launch angles

1. **(c)** $55°$**.** The angles are complementary: $35° + 55° = 90°$. Their doubled angles are supplementary, so the sine factor in the range formula is equal.