Projectile Range

Course Author

From launch to landing

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- Its horizontal component is $v_0 \cos(\theta)$, its vertical component $v_0 \sin(\theta)$
- The Gravitational acceleration is vertical, so the horizontal component never changes

The range takes three steps to derive

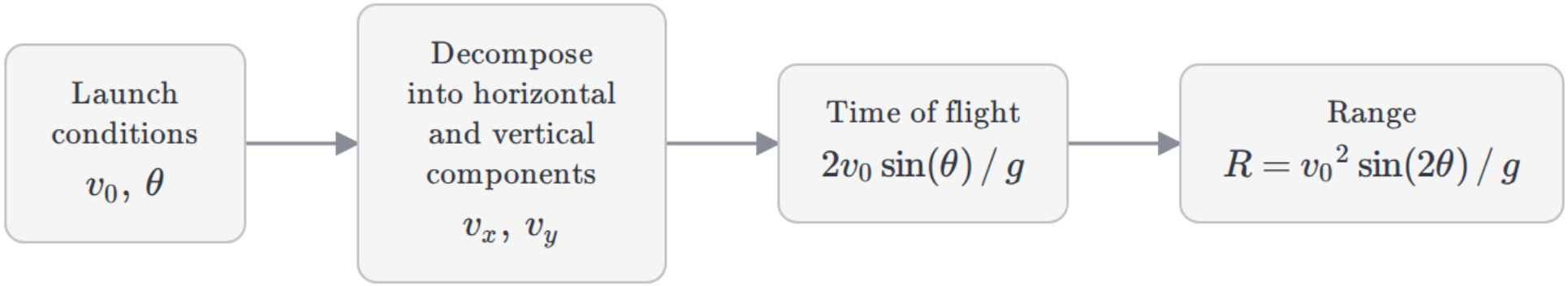


Diagram 2.1.1 Steps of the range derivation. From the launch conditions to the range, through the two velocity components and the time of flight.

The flight ends when the height is zero again

$$y(t) = v_0 \sin(\theta)\, t - \tfrac{1}{2}\, g\, t^2 = 0$$

y Vertical coordinate t Time

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The Time of landing is $2v_0 \sin(\theta)/g$.

Range is horizontal speed times flight time

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$$\begin{aligned} R &= v_0 \cos(\theta) \cdot \frac{2v_0 \sin(\theta)}{g} \\ &= \frac{{v_0}^2 \sin(2\theta)}{g} \quad \text{double-angle identity} \end{aligned} \tag{2.1.1}$$

The range is largest at a 45° launch angle

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- $\sin(2\theta)$ is largest at $\theta = 45^\circ$
- The angles θ and $90^\circ - \theta$ give the same range

Try it

Watch the range as the angle sweeps

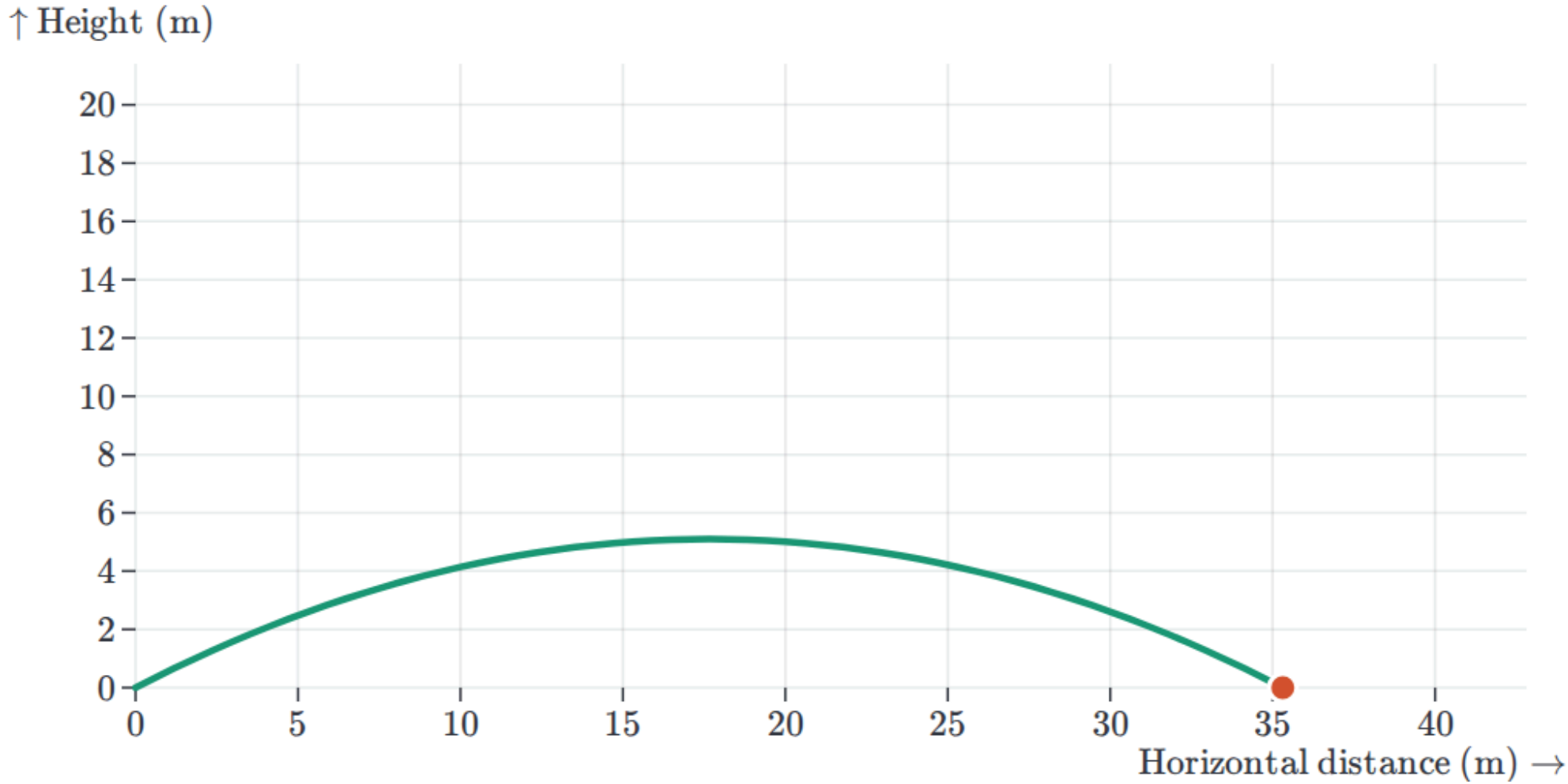


Figure 2.1.1 Projectile trajectory explorer. The path from launch to landing; the dot marks the landing point. Shown for $v_0 = 20$ m/s and $\theta = 30°$: range 35.3 m, time of flight 2.04 s.

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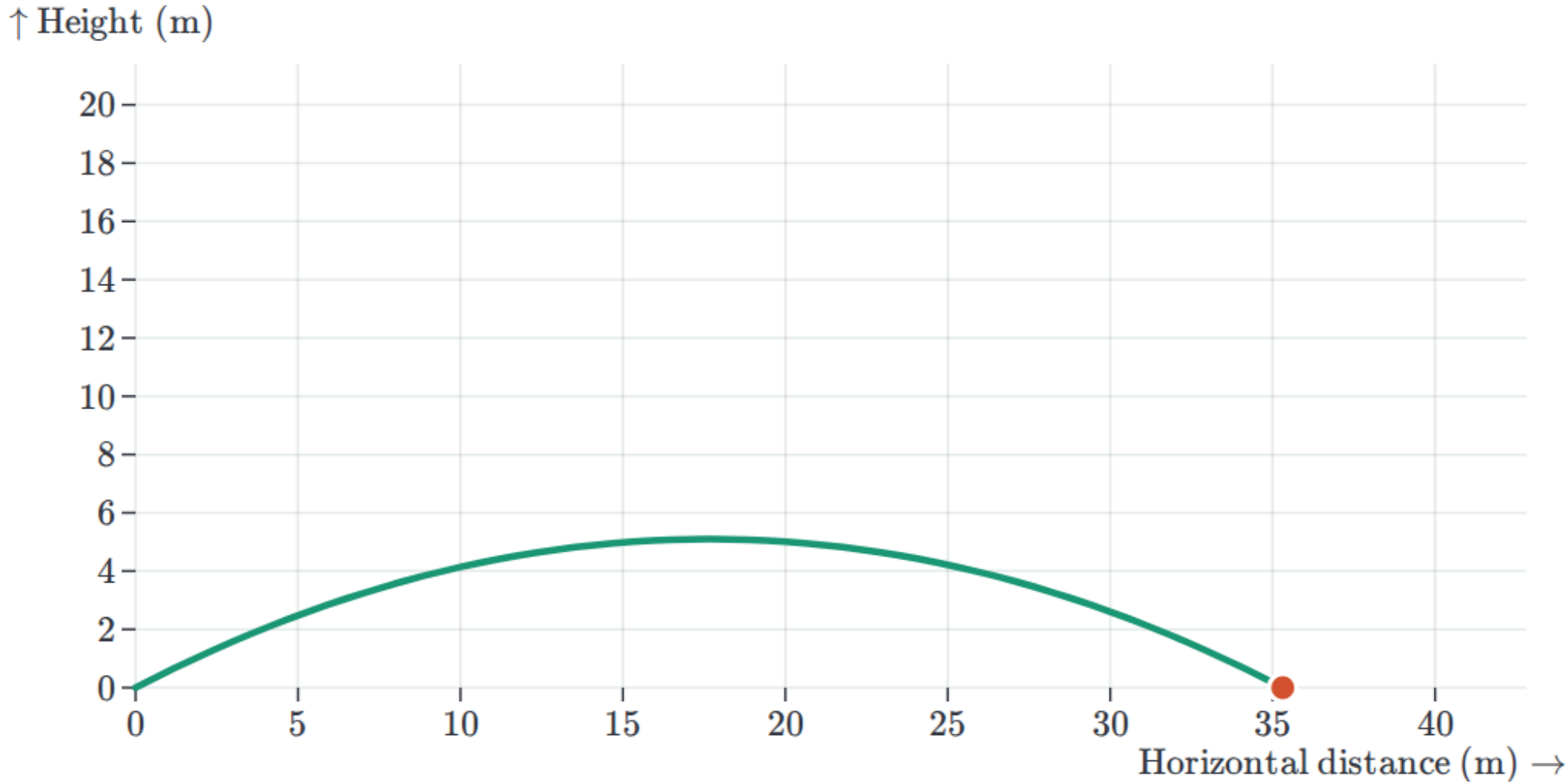


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From a height

A launch from a height flies farther

$$R = \frac{v_0 \cos(\theta)}{g}\left[v_0 \sin(\theta) + \sqrt{(v_0 \sin(\theta))^2 + 2gh}\right] \tag{2.1.3}$$

- With Launch height $h = 0$ it gives the range formula again

Check

Compute a range from a speed and an angle

Knowledge check 2.1.1: Projectile range

1. A ball is launched over level ground at 20 m/s, 30° above the horizontal. Using $g = 9.81$ m/s^2 and ignoring air resistance, what is its range in meters?

What you can now do

- Given a launch speed and angle over level ground, compute the horizontal range of a projectile.

Notation

The symbols of this deck, in the order they first appear.

v_0 **Launch speed**. The object's speed at the start of the scenario being analyzed — its initial velocity in a straight-line problem, or its launch speed the instant a projectile leaves the ground. *Units:* meters per second.

θ **Launch angle**. The angle between a projectile's initial velocity and the horizontal ground at launch. *Units:* degrees above the horizontal.

g **Gravitational acceleration**. The magnitude of gravitational acceleration, treated as constant near the ground. *Units:* meters per second squared.

$v(t)$ **Velocity**. The rate at which position changes with time. It is a vector; in a straight-line problem only its size and sign matter, and that is what the scalar v(t) tracks. *Units:* meters per second.

$$\boldsymbol{v} = \frac{\mathrm{d}\,\boldsymbol{r}}{\mathrm{d}t}\,; \quad v(t) = v_0 + a\,t$$

$\square_x$ **Horizontal component**. The part of a vector quantity along the horizontal axis, positive in the direction of flight.

$\square_y$ **Vertical component**. The part of a vector quantity along the vertical axis, positive upward.

R **Range**. Total horizontal distance a projectile covers before it returns to its launch height. *Units:* meters.

$$R = \frac{{v_0}^2 \sin(2\theta)}{g}$$

y **Vertical coordinate**. The signed distance from the origin along the vertical axis, measured positive upward. *Units:* meters.

t **Time**. Elapsed time since the object was launched or released, measured in seconds. *Units:* seconds after launch.

h **Launch height**. The height of the launch point above the plane on which the projectile lands. It is zero for level ground. *Units:* meters.