Projectile Range

Explico Playground — Motion

Course Author

Content unchanged since version 1.0.0

From launch to landing

Gravity changes only the vertical motion

- The launch velocity has a Launch speed and a Launch angle above the horizontal
- Its horizontal component is $v_0 \cos(\theta)$, its vertical component $v_0 \sin(\theta)$

v_0 Launch speed θ Launch angle g Gravitational acceleration

Gravity changes only the vertical motion

- The launch velocity has a Launch speed and a Launch angle above the horizontal
- Its horizontal component is $v_0 \cos(\theta)$, its vertical component $v_0 \sin(\theta)$
- The Gravitational acceleration is vertical, so the horizontal component never changes

v_0 Launch speed θ Launch angle g Gravitational acceleration

The range takes three steps to derive

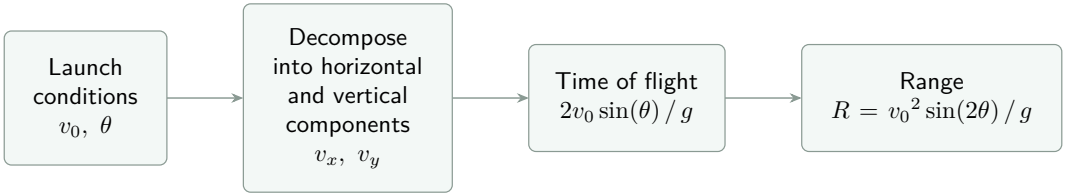


Diagram 2.1.1 **Steps of the range derivation.** From the launch conditions to the range, through the two velocity components and the time of flight.

The flight ends when the height is zero again

$$y(t) = v_0 \sin(\theta)\, t - \tfrac{1}{2}\, g\, t^2 = 0$$

y Vertical coordinate t Time

The flight ends when the height is zero again

$$y(t) = v_0 \sin(\theta)\, t - \tfrac{1}{2}\, g\, t^2 = 0$$

The Time of landing is $2v_0 \sin(\theta)/g$.

y Vertical coordinate t Time

Range is horizontal speed times flight time

$$R = v_0 \cos(\theta) \cdot \frac{2v_0 \sin(\theta)}{g}$$

R Range

Range is horizontal speed times flight time

$$R = v_0 \cos(\theta) \cdot \frac{2v_0 \sin(\theta)}{g}$$

$$\begin{aligned} R &= v_0 \cos(\theta) \cdot \frac{2v_0 \sin(\theta)}{g} \\ &= \frac{{v_0}^2 \sin(2\theta)}{g} \quad \text{double-angle identity} \end{aligned} \tag{2.1.1}$$

The range is largest at a 45° launch angle

$$
\begin{aligned}
R &= v_0 \cos(\theta) \cdot \frac{2v_0 \sin(\theta)}{g} \\
&= \frac{{v_0}^2 \sin(2\theta)}{g} \quad \text{double-angle identity}
\end{aligned}
\tag{2.1.1}
$$

- $\sin(2\theta)$ is largest at $\theta = 45^\circ$

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&= \frac{{v_0}^2 \sin(2\theta)}{g} \quad \text{double-angle identity}
\end{aligned}
\tag{2.1.1}
$$

- $\sin(2\theta)$ is largest at $\theta = 45^\circ$
- The angles θ and $90^\circ - \theta$ give the same range

Try it

Watch the range as the angle sweeps

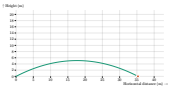


Figure 2.1.1 **Projectile trajectory explorer.** The path from launch to landing; the dot marks the landing point. Shown for $v_0 = 20$ m/s and $\theta = 30°$: range 35.3 m, time of flight 2.04 s.

From a height

A launch from a height flies farther

$$R = \frac{v_0 \cos(\theta)}{g} \left[v_0 \sin(\theta) + \sqrt{(v_0 \sin(\theta))^2 + 2gh} \right] \tag{2.1.3}$$

- With Launch height $h = 0$ it gives the range formula again

h Launch height

Check

Compute a range from a speed and an angle

Knowledge check 2.1.1 Projectile range

1. A ball is launched over level ground at 20 m/s, 30° above the horizontal. Using $g = 9.81$ m/s^2 and ignoring air resistance, what is its range in meters?

What you can now do

- Given a launch speed and angle over level ground, compute the horizontal range of a projectile.

Notation

v_0 **Launch speed**. The object's speed at the start of the scenario being analyzed — its initial velocity in a straight-line problem, or its launch speed the instant a projectile leaves the ground. *Units:* meters per second.

θ **Launch angle**. The angle between a projectile's initial velocity and the horizontal ground at launch. *Units:* degrees above the horizontal.

g **Gravitational acceleration**. The magnitude of gravitational acceleration, treated as constant near the ground. *Units:* meters per second squared.

$v(t)$ **Velocity**. The rate at which position changes with time. It is a vector; in a straight-line problem only its size and sign matter, and that is what the scalar v(t) tracks. *Units:* meters per second. $\mathbf{v} = \dfrac{\mathrm{d}\,\mathbf{r}}{\mathrm{d}t}\,; \quad v(t) = v_0 + a\,t$

$\square_x$ **Horizontal component**. The part of a vector quantity along the horizontal axis, positive in the direction of flight.

Notation (continued)

$\square_y$ **Vertical component**. The part of a vector quantity along the vertical axis, positive upward.

R **Range**. Total horizontal distance a projectile covers before it returns to its launch height. *Units:* meters. $R = \dfrac{{v_0}^2 \sin(2\theta)}{g}$

y **Vertical coordinate**. The signed distance from the origin along the vertical axis, measured positive upward. *Units:* meters.

t **Time**. Elapsed time since the object was launched or released, measured in seconds. *Units:* seconds after launch.

h **Launch height**. The height of the launch point above the plane on which the projectile lands. It is zero for level ground. *Units:* meters.