The Optimal Launch Angle

Course Author

The angle that flies farthest

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- So the range is largest at $\theta = 45^\circ$

The search fixes the speed and tries each angle

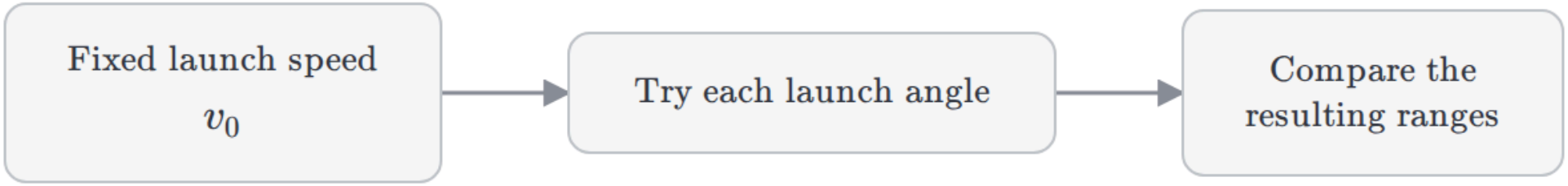


Diagram 2.3.1 Inputs to the launch-angle search. The launch speed stays fixed while each launch angle is tried in turn, and the ranges they give are compared.

A maximum without calculus

A square turns the product into a polynomial

$$\begin{aligned}\sin^2(\theta)\cos^2(\theta) &= \sin^2(\theta)(1-\sin^2(\theta)) \\ &= u - u^2\end{aligned}$$

- The variable is the Squared sine of the launch angle, $u = \sin^2(\theta)$

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$$\begin{aligned} \theta^* &= \arcsin\left(\sqrt{u}\right) && \text{definition of } u \text{, with } \sin(\theta) \geq 0 \\ &= \arcsin\left(\tfrac{1}{\sqrt{2}}\right) && \text{vertex at } u = \tfrac{1}{2} \\ &= 45^\circ \end{aligned} \tag{2.3.2}$$

The polynomial is a downward parabola

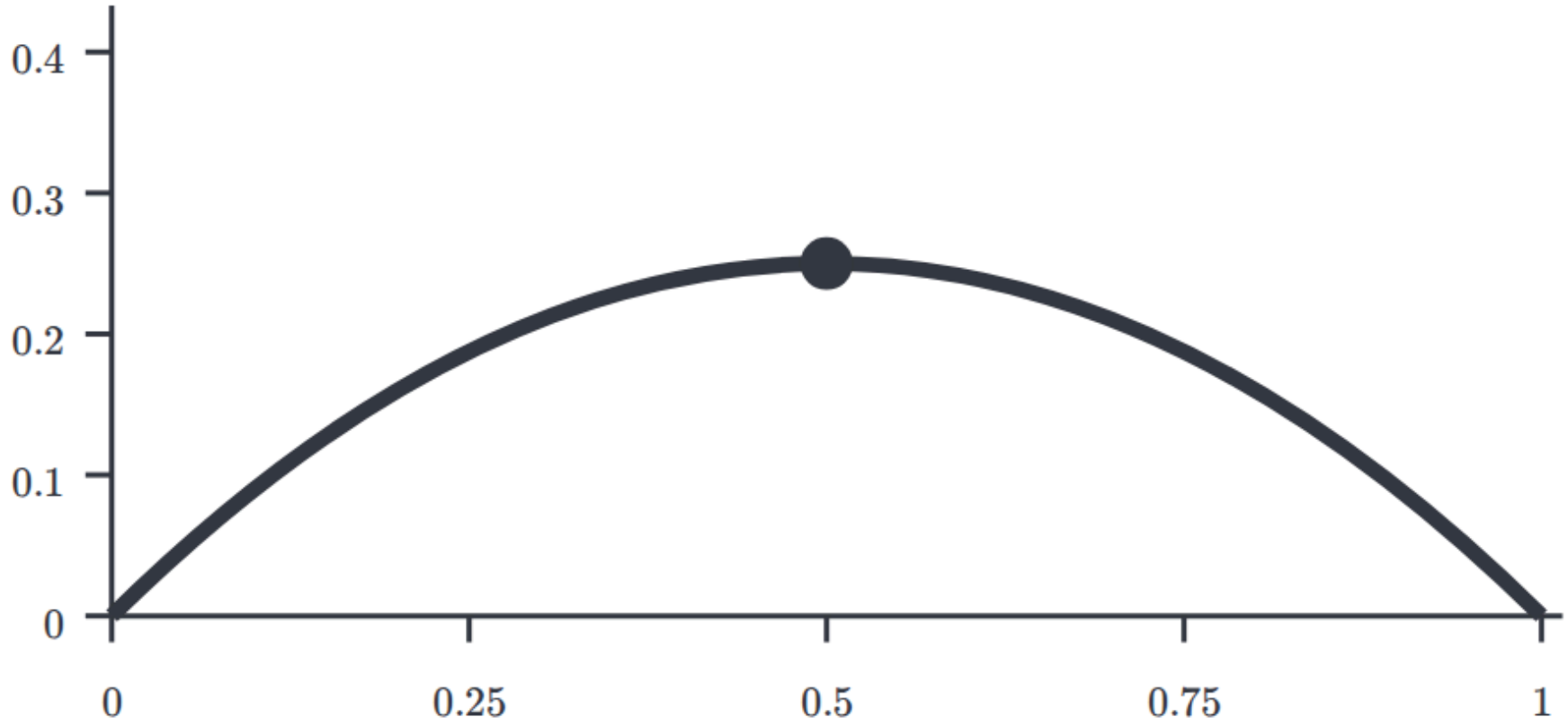


Figure 2.3.2 The polynomial is a downward parabola. From 0 at input 0 it rises to its maximum, 0.25, at input 0.5, and falls symmetrically back to 0 at input 1.

Complementary angles

Complementary angles give equal ranges

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- $\sin(2\theta') = \sin(180^\circ - 2\theta) = \sin(2\theta)$

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- $\sin(2\theta') = \sin(180^\circ - 2\theta) = \sin(2\theta)$
- So launches at 30° and 60° land at the same place

The range curve is symmetric about 45°

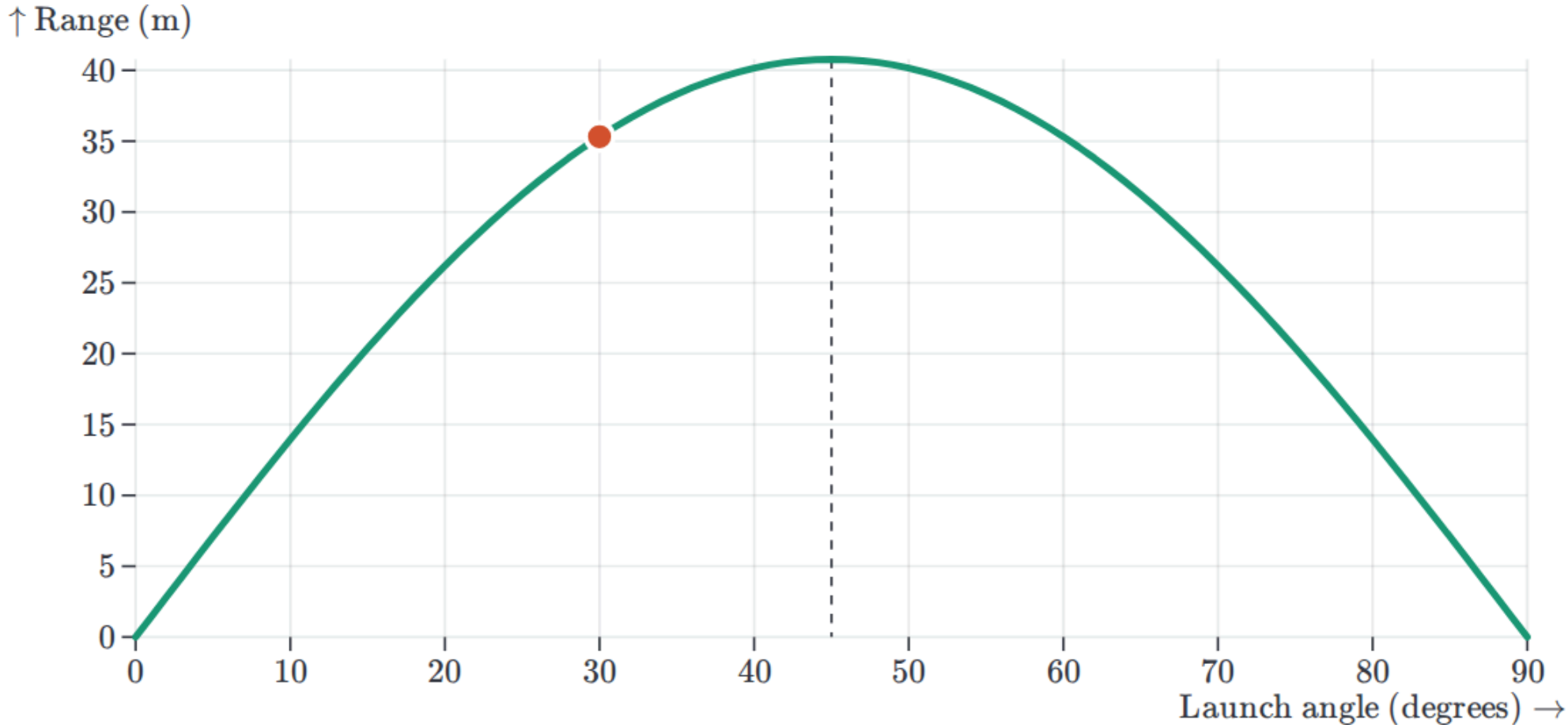


Figure 2.3.1 Range vs. launch angle explorer. The range at every launch angle for one launch speed; the dashed line marks 45°, where the range is largest. Shown for $v_0 = 20$ m/s; the dot marks $\theta = 30°$: range 35.3 m, 5.5 m short of the maximum (40.8 m, at 45°).

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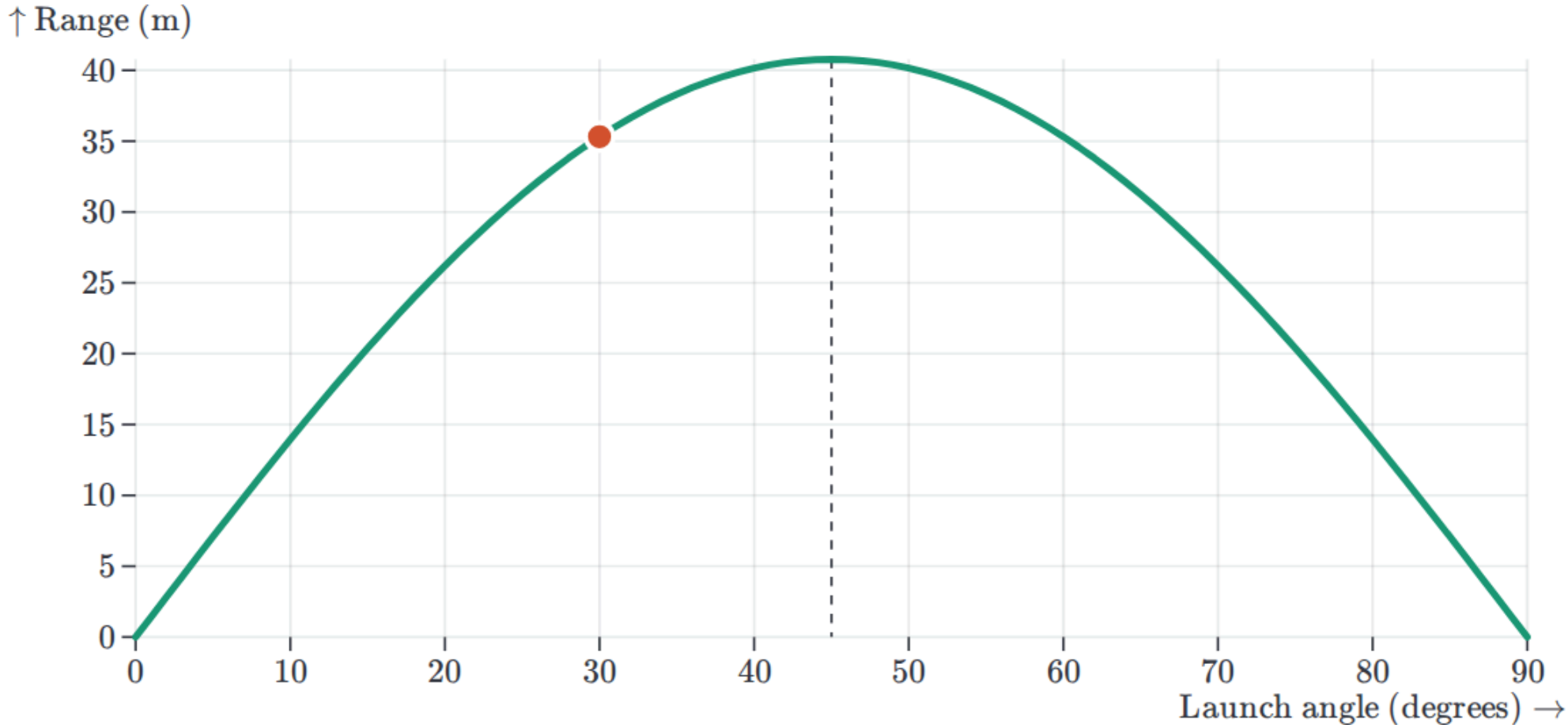


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Check

Find the best launch angle

Knowledge check 2.3.1: Optimal launch angle

1. For a fixed launch speed over level ground, with no air resistance, which launch angle maximizes range?
   - (a) 30°
   - (b) 45°
   - (c) 60°
   - (d) 90°

What you can now do

- For a fixed launch speed over level ground, identify the launch angle that maximizes range.

Notation

The symbols of this deck, in the order they first appear.

v_0 **Launch speed**. The object's speed at the start of the scenario being analyzed — its initial velocity in a straight-line problem, or its launch speed the instant a projectile leaves the ground. *Units:* meters per second.

R **Range**. Total horizontal distance a projectile covers before it returns to its launch height. *Units:* meters.

$$R = \frac{{v_0}^2 \sin(2\theta)}{g}$$

θ **Launch angle**. The angle between a projectile's initial velocity and the horizontal ground at launch. *Units:* degrees above the horizontal.

u **Squared sine of the launch angle**. The squared sine of the launch angle, used as the variable in the range polynomial. *Units:* dimensionless.

$$u = \sin^2(\theta)$$

θ^* **Optimal launch angle**. The launch angle that maximizes range for a given fixed launch speed, over level ground. *Units:* degrees above the horizontal.

θ' **Complementary launch angle**. The launch angle whose sum with a given launch angle is 90 degrees. *Units:* degrees above the horizontal.