The Optimal Launch Angle

Explico Playground — Motion

Course Author

Content with changes after version 1.0.0, not yet released

The angle that flies farthest

The angle enters the range only through a sine

- At a fixed Launch speed, the Range changes with the Launch angle only through $\sin(2\theta)$
- From 0° to 180°, the sine is largest at 90°

v_0 Launch speed R Range θ Launch angle

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- From 0° to 180°, the sine is largest at 90°
- So the range is largest at $\theta = 45^\circ$

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The search fixes the speed and tries each angle

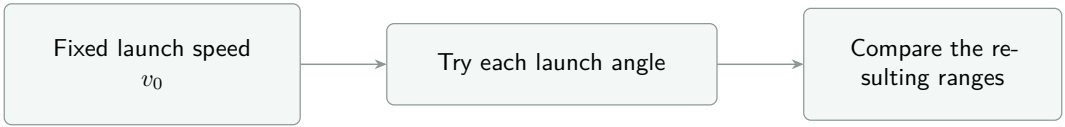


Diagram 2.3.1 **Inputs to the launch-angle search.** The launch speed stays fixed while each launch angle is tried in turn, and the ranges they give are compared.

A maximum without calculus

A square turns the product into a polynomial

$$\begin{aligned}\sin^2(\theta)\,\cos^2(\theta) &= \sin^2(\theta)\left(1-\sin^2(\theta)\right)\\ &= u - u^2\end{aligned}$$

- The variable is the Squared sine of the launch angle, $u = \sin^2(\theta)$

u Squared sine of the launch angle

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- Its roots are $u = 0$ and $u = 1$, so it peaks at $u = \frac{1}{2}$

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$$\begin{aligned} \theta^* &= \arcsin\left(\sqrt{u}\right) && \text{definition of } u, \text{ with } \sin(\theta) \geq 0 \\ &= \arcsin\left(\tfrac{1}{\sqrt{2}}\right) && \text{vertex at } u = \tfrac{1}{2} \\ &= 45^\circ \end{aligned} \tag{2.3.2}$$

θ^* Optimal launch angle

The polynomial is a downward parabola

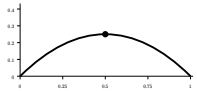


Figure 2.3.2 The polynomial is a downward parabola. From 0 at input 0 it rises to its maximum, 0.25, at input 0.5, and falls symmetrically back to 0 at input 1.

Complementary angles

Complementary angles give equal ranges

- The Complementary launch angle of θ is $\theta' = 90° - \theta$
- $\sin\left(2\theta'\right) = \sin\left(180° - 2\theta\right) = \sin(2\theta)$

θ' Complementary launch angle

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- $\sin\left(2\theta'\right) = \sin\left(180^\circ - 2\theta\right) = \sin(2\theta)$
- So launches at 30° and 60° land at the same place

θ' Complementary launch angle

The range curve is symmetric about $45°$

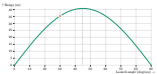


Figure 2.3.1 **Range vs. launch angle explorer.** The range at every launch angle for one launch speed; the dashed line marks 45°, where the range is largest. Shown for $v_0 = 20$ m/s; the dot marks $\theta = 30°$: range 35.3 m, 5.5 m short of the maximum (40.8 m, at 45°).

Check

Find the best launch angle

Knowledge check 2.3.1 Optimal launch angle

1. For a fixed launch speed over level ground, with no air resistance, which launch angle maximizes range?
   (a) $30°$
   (b) $45°$
   (c) $60°$
   (d) $90°$

What you can now do

- For a fixed launch speed over level ground, identify the launch angle that maximizes range.

Notation

v_0 **Launch speed**. The object's speed at the start of the scenario being analyzed — its initial velocity in a straight-line problem, or its launch speed the instant a projectile leaves the ground. *Units:* meters per second.

R **Range**. Total horizontal distance a projectile covers before it returns to its launch height. *Units:* meters. $R = \dfrac{{v_0}^2 \sin(2\theta)}{g}$

θ **Launch angle**. The angle between a projectile's initial velocity and the horizontal ground at launch. *Units:* degrees above the horizontal.

u **Squared sine of the launch angle**. The squared sine of the launch angle, used as the variable in the range polynomial. *Units:* dimensionless. $u = \sin^2(\theta)$

θ^* **Optimal launch angle**. The launch angle that maximizes range for a given fixed launch speed, over level ground. *Units:* degrees above the horizontal.

Notation (continued)

θ' **Complementary launch angle**. The launch angle whose sum with a given launch angle is 90 degrees. *Units:* degrees above the horizontal.