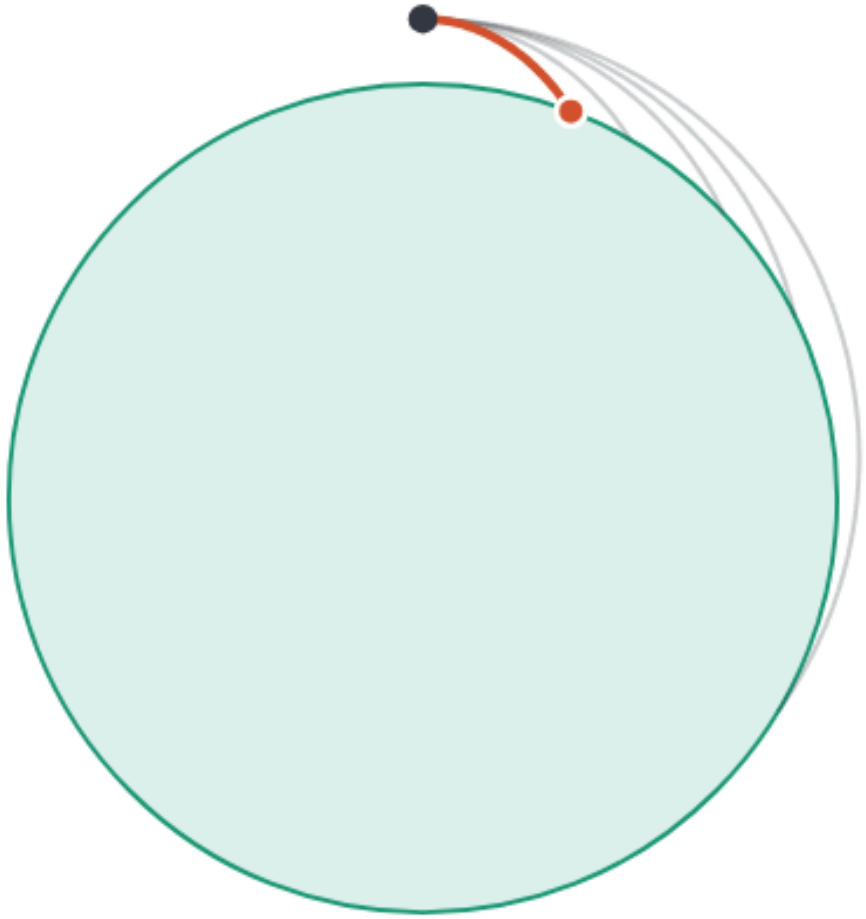
Newton's Cannonball

Course Author

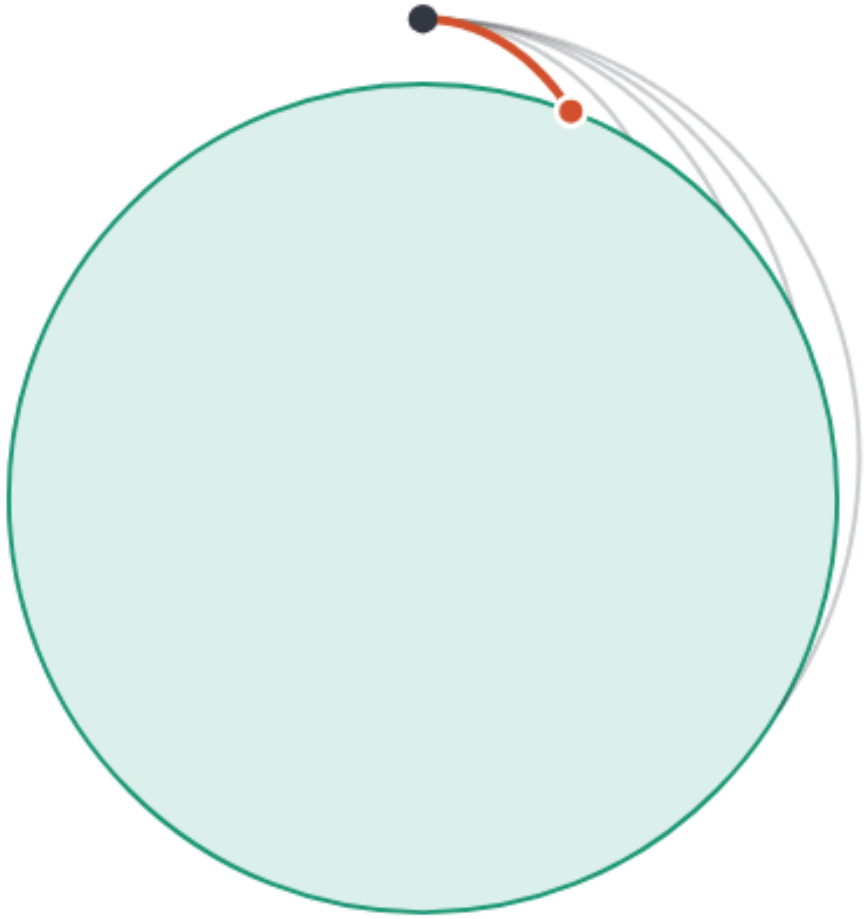
A cannon above the air

Speed alone decides: land, orbit or escape



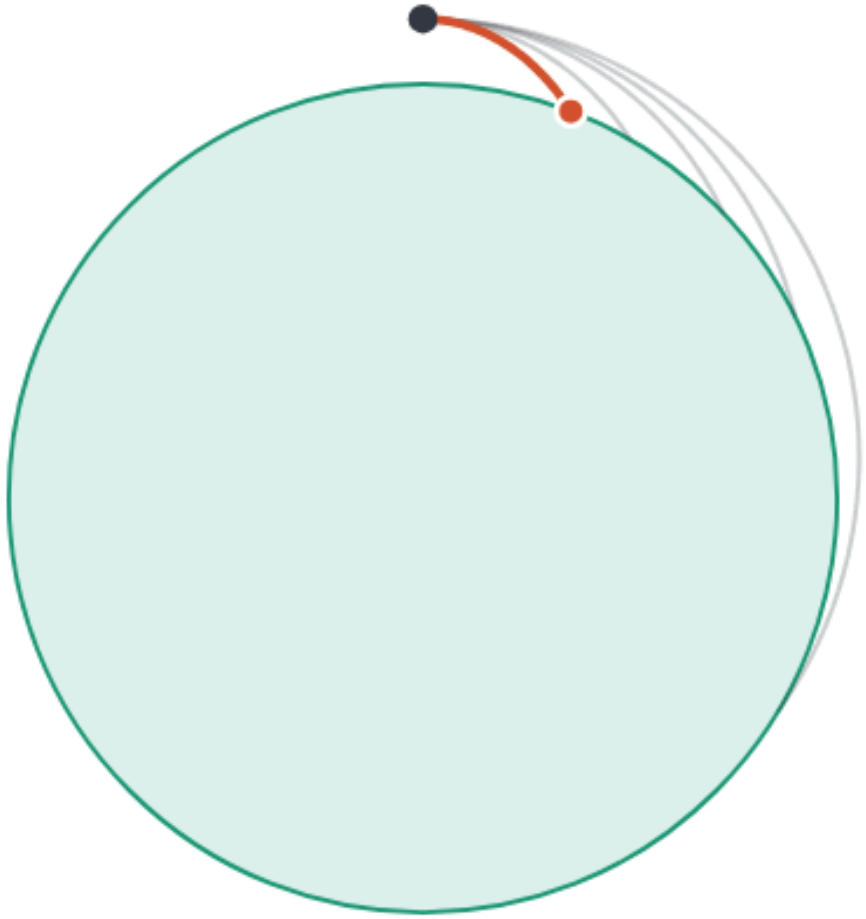
Newton's cannonball. A cannon on a mountain 1000 km high fires a ball horizontally; the faint paths are slower shots. Shown for $v_0 = 4.00$ km/s: the ball lands 2,327 km away, 21° around the Earth.

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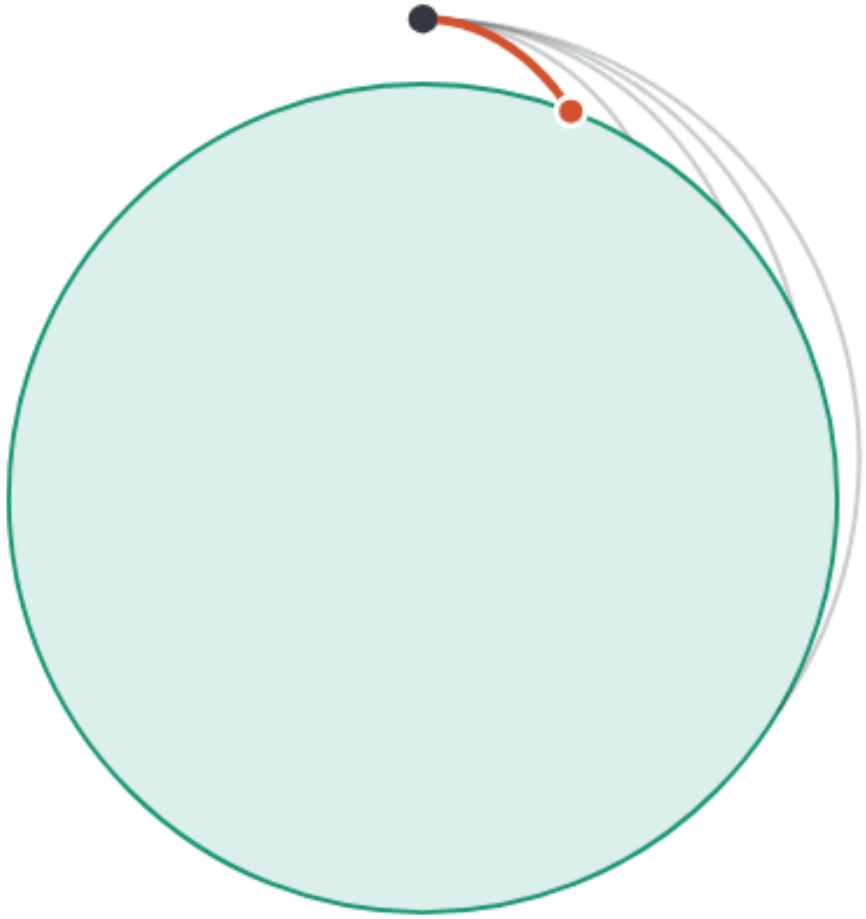
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The speed of a low orbit

The ground curves away as fast as the ball falls

- In a time t the ball falls $\frac{1}{2}\,g\,t^2$
- In that time the ground drops $(v_1\,t)^2/(2R_E)$ below the horizontal

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$$v_1 = \sqrt{g\, R_E} \tag{1}$$

A ground-skimming orbit needs 7.9 km/s

$$v_1 = \sqrt{9.81\ \mathrm{m/s^2} \times 6.371 \times 10^6\ \mathrm{m}} \approx 7.9 \times 10^3\ \mathrm{m/s}$$

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- That is about 28 000 km/h
- From the lab's mountain, 1000 km high, less: 7.35 km/s

One lap of a low orbit takes 84 min

$$T = \frac{2\pi\, R_E}{v_1} \approx \frac{4.0 \times 10^7 \text{ m}}{7.9 \times 10^3 \text{ m/s}} \approx 5.1 \times 10^3 \text{ s}$$

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- The space station, about 400 km up, takes about 92 min

Faster than a circle

At $\sqrt{2}$ times the circle's speed, it escapes

$$v_2 = \sqrt{2}\, v_1 \approx 11.2 \text{ km/s}$$

- Between v_1 and v_2, the ball orbits on an ellipse

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$$v_2 = \sqrt{2}\, v_1 \approx 11.2 \text{ km/s}$$

- Between v_1 and v_2, the ball orbits on an ellipse
- At v_2 or faster, it never comes back

The Moon is a cannonball too

The Moon falls toward the Earth all the time

- At 60 R_E from the center, the Moon moves sideways at about 1 km/s
- Like the cannonball, it falls toward the Earth and never lands

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$$a = \frac{4\pi^2\, r}{T^2} \approx 2.7 \times 10^{-3}\ \mathrm{m/s^2} \approx \frac{g}{60^2}$$

Check

Compute the speed of a low orbit

Knowledge check 2.4.1: The speed of a low orbit

1. A ball circles the Earth just above the ground, with no air to slow it. Using $g = 9.81$ m/s^2 and $R_E = 6.37 \times 10^6$ m, what is its speed in km/s?

Weigh gravity at the Moon’s distance

Knowledge check 2.4.2: Gravity at the Moon's distance

1. The Moon orbits about 60 Earth radii from the Earth's center. How large is the Earth's gravitational acceleration there, compared with g at the ground?
   - (a) The same as at the ground
   - (b) One sixtieth of it
   - (c) One thirty-six-hundredth of it
   - (d) Zero: the Moon is weightless

What you can now do

- Explain an orbit as a horizontal launch so fast that the ground curves away as fast as the projectile falls, and compute the speed and the period of a low orbit.

References

References

- [1] Newton, A Treatise of the System of the World (1728).
- [2] Ling et al., University Physics Volume 1 (2016). ch. 13, “Satellite Orbits and Energy”. https://openstax.org/details/books/university-physics-volume-1
- [3] Ling et al., University Physics Volume 1 (2016). ch. 13, “Gravitational Potential Energy and Total Energy”. https://openstax.org/details/books/university-physics-volume-1

Notation

The symbols of this deck, in the order they first appear.

v_0 **Launch speed**. The object's speed at the start of the scenario being analyzed — its initial velocity in a straight-line problem, or its launch speed the instant a projectile leaves the ground. *Units:* meters per second.

t **Time**. Elapsed time since the object was launched or released, measured in seconds. *Units:* seconds after launch.

g **Gravitational acceleration**. The magnitude of gravitational acceleration, treated as constant near the ground. *Units:* meters per second squared.

v_1 **Circular orbital speed**. The horizontal launch speed at which a projectile falls around the Earth on a circle and never reaches the ground. *Units:* meters per second.

R_E **Earth's radius**. The distance from the Earth's center to its surface. *Units:* meters.

T **Orbital period**. The time a body on a closed orbit takes to come back to where it started. *Units:* seconds.

v_2 **Escape speed**. The launch speed from which a projectile leaves the Earth and never comes back. *Units:* meters per second.

$\boldsymbol{a}$ **Acceleration**. The rate at which velocity changes with time. In this course it is held constant over a scenario, so it is the same vector at every instant. *Units:* meters per second squared.

$$\boldsymbol{a} = \frac{\mathrm{d}\,\boldsymbol{v}}{\mathrm{d}t}$$

r **Orbit radius**. The distance from the Earth's center to a body that orbits it on a circle. *Units:* meters.