Newton's Cannonball

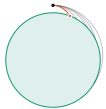
Explico Playground — Motion

Course Author

Content unchanged since version 1.0.0

A cannon above the air

Speed alone decides: land, orbit or escape



Newton's cannonball. A cannon on a mountain 1000 km high fires a ball horizontally; the faint paths are slower shots. Shown for $v_0 = 4.00$ km/s: the ball lands 2,327 km away, 21° around the Earth.

The speed of a low orbit

The ground curves away as fast as the ball falls

- In a time t the ball falls $\frac{1}{2}\,g\,t^2$
- In that time the ground drops $(v_1\,t)^2/(2R_E)$ below the horizontal

t Time g Gravitational acceleration v_1 Circular orbital speed R_E Earth's radius

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$$v_1 = \sqrt{g\, R_E} \tag{1}$$

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A ground-skimming orbit needs 7.9 km/s

$$v_1 = \sqrt{9.81\ \mathsf{m/s}^2 \times 6.371 \times 10^6\ \mathsf{m}} \approx 7.9 \times 10^3\ \mathsf{m/s}$$

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- That is about $28\,000$ km/h
- From the lab's mountain, 1000 km high, less: 7.35 km/s

One lap of a low orbit takes 84 min

$$T = \frac{2\pi\, R_E}{v_1} \approx \frac{4.0 \times 10^7\ \mathsf{m}}{7.9 \times 10^3\ \mathsf{m/s}} \approx 5.1 \times 10^3\ \mathsf{s}$$

T Orbital period

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- The space station, about 400 km up, takes about 92 min

T Orbital period

Faster than a circle

At $\sqrt{2}$ times the circle's speed, it escapes

$$v_2 = \sqrt{2}\, v_1 \approx 11.2 \text{ km/s}$$

- Between v_1 and v_2, the ball orbits on an ellipse

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- Between v_1 and v_2, the ball orbits on an ellipse
- At v_2 or faster, it never comes back

The Moon is a cannonball too

The Moon falls toward the Earth all the time

- At $60\,R_E$ from the center, the Moon moves sideways at about 1 km/s
- Like the cannonball, it falls toward the Earth and never lands

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$$a = \frac{4\pi^2\, r}{T^2} \approx 2.7 \times 10^{-3}\ \text{m/s}^2 \approx \frac{g}{60^2}$$

a Acceleration r Orbit radius

Check

Compute the speed of a low orbit

Knowledge check 2.4.1 The speed of a low orbit

1. A ball circles the Earth just above the ground, with no air to slow it. Using $g = 9.81$ m/s^2 and $R_E = 6.37 \times 10^6$ m, what is its speed in km/s?

Weigh gravity at the Moon's distance

Knowledge check 2.4.2 Gravity at the Moon's distance

1. The Moon orbits about 60 Earth radii from the Earth's center. How large is the Earth's gravitational acceleration there, compared with g at the ground?
   (a) The same as at the ground
   (b) One sixtieth of it
   (c) One thirty-six-hundredth of it
   (d) Zero: the Moon is weightless

What you can now do

- Explain an orbit as a horizontal launch so fast that the ground curves away as fast as the projectile falls, and compute the speed and the period of a low orbit.

References

[1] Newton, A Treatise of the System of the World (1728).

[2] Ling et al., University Physics Volume 1 (2016). ch. 13, “Satellite Orbits and Energy”. https://openstax.org/details/books/university-physics-volume-1

[3] Ling et al., University Physics Volume 1 (2016). ch. 13, “Gravitational Potential Energy and Total Energy”. https://openstax.org/details/books/university-physics-volume-1

Notation

v_0 **Launch speed**. The object's speed at the start of the scenario being analyzed — its initial velocity in a straight-line problem, or its launch speed the instant a projectile leaves the ground. *Units:* meters per second.

t **Time**. Elapsed time since the object was launched or released, measured in seconds. *Units:* seconds after launch.

g **Gravitational acceleration**. The magnitude of gravitational acceleration, treated as constant near the ground. *Units:* meters per second squared.

v_1 **Circular orbital speed**. The horizontal launch speed at which a projectile falls around the Earth on a circle and never reaches the ground. *Units:* meters per second.

R_E **Earth's radius**. The distance from the Earth's center to its surface. *Units:* meters.

Notation (continued)

T **Orbital period**. The time a body on a closed orbit takes to come back to where it started. *Units:* seconds.

v_2 **Escape speed**. The launch speed from which a projectile leaves the Earth and never comes back. *Units:* meters per second.

$\mathbf{a}$ **Acceleration**. The rate at which velocity changes with time. In this course it is held constant over a scenario, so it is the same vector at every instant. *Units:* meters per second squared. $\mathbf{a} = \dfrac{\mathrm{d}\,\mathbf{v}}{\mathrm{d}t}$

r **Orbit radius**. The distance from the Earth's center to a body that orbits it on a circle. *Units:* meters.