Newton's Cannonball

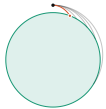
Explico Playground — Motion

Course Author

Content unchanged since version 1.0.0

A cannon above the air

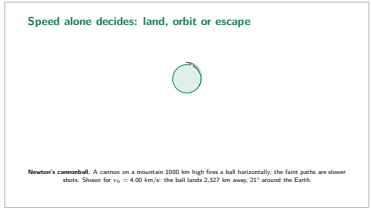
Speed alone decides: land, orbit or escape



Newton's cannonball. A cannon on a mountain 1000 km high fires a ball horizontally; the faint paths are slower shots. Shown for $v_0 = 4.00$ km/s: the ball lands 2,327 km away, 21° around the Earth.

Slide 3

Speed alone decides: land, orbit or escape



Newton drew this cannon on a mountain as tall [1]. Fired faster, the ball lands farther around the Earth. At 7.35 km/s the ground curves away as fast as the ball falls, so the ball never lands. Faster still, it climbs on an ellipse and comes back. From 10.4 km/s on, it leaves for good.

The speed of a low orbit

The ground curves away as fast as the ball falls

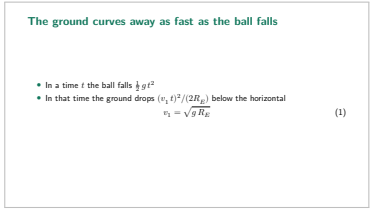
- In a time t the ball falls $\frac{1}{2}\, g\, t^2$
- In that time the ground drops $(v_1\, t)^2/(2R_E)$ below the horizontal

$$v_1 = \sqrt{g\, R_E} \tag{1}$$

t Time g Gravitational acceleration v_1 Circular orbital speed R_E Earth's radius

Slide 5

The ground curves away as fast as the ball falls



In a time t the Gravitational acceleration pulls the ball down by $\frac{1}{2}\, g\, t^2$, as for any projectile. In the same time the ball moves $v_1\, t$ along the horizontal line from the cannon. The round ground falls away below that line by the square of that distance over twice the Earth's radius. When the two drops are equal at every moment, the ball comes no closer to the ground. Setting them equal gives the Circular orbital speed [2].

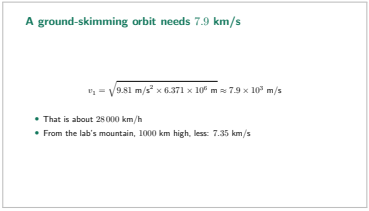
A ground-skimming orbit needs 7.9 km/s

$$v_1 = \sqrt{9.81\ \text{m/s}^2 \times 6.371 \times 10^6\ \text{m}} \approx 7.9 \times 10^3\ \text{m/s}$$

- That is about $28\,000$ km/h
- From the lab's mountain, 1000 km high, less: 7.35 km/s

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A ground-skimming orbit needs 7.9 km/s



Put in the numbers for the ground. Gravity is weaker 1000 km up, so the circle there needs less speed. A rocket first reached this speed in 1957, with Sputnik 1.

One lap of a low orbit takes 84 min

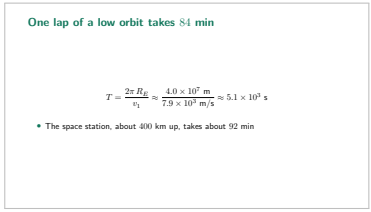
$$T = \frac{2\pi\, R_E}{v_1} \approx \frac{4.0 \times 10^7 \text{ m}}{7.9 \times 10^3 \text{ m/s}} \approx 5.1 \times 10^3 \text{ s}$$

- The space station, about 400 km up, takes about 92 min

T Orbital period

Slide 7

One lap of a low orbit takes 84 min



One lap is the circumference of the Earth at the circular speed. A higher orbit is longer and a little slower, so its Orbital period is longer. The space station's lap is the same physics.

Faster than a circle

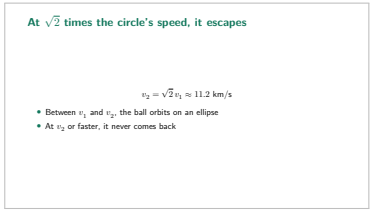
At $\sqrt{2}$ times the circle's speed, it escapes

$$v_2 = \sqrt{2}\, v_1 \approx 11.2 \text{ km/s}$$

- Between v_1 and v_2, the ball orbits on an ellipse
- At v_2 or faster, it never comes back

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At $\sqrt{2}$ times the circle's speed, it escapes



Faster than the circle, the ball climbs as it goes round, slows down, and falls back to the cannon on an ellipse. At the Escape speed the ball has enough kinetic energy to climb out of the Earth's gravity for good. The escape speed is $\sqrt{2}$ times the circular speed at the same height [3].

The Moon is a cannonball too

The Moon falls toward the Earth all the time

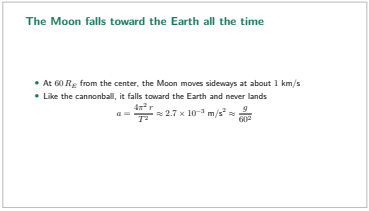
- At $60\,R_E$ from the center, the Moon moves sideways at about 1 km/s
- Like the cannonball, it falls toward the Earth and never lands

$$a = \frac{4\pi^2\, r}{T^2} \approx 2.7 \times 10^{-3}\ \text{m/s}^2 \approx \frac{g}{60^2}$$

$\mathbf{a}$ Acceleration r Orbit radius

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The Moon falls toward the Earth all the time



Newton compared the Moon with the cannonball. The Moon's Orbit radius is about 3.84×10^8 m, and its period is about 27.3 d. On a circle of radius r run once every T, the acceleration toward the center is $4\pi^2\, r/T^2$. It is 3600 times smaller than g, and $3600 = 60^2$. So gravity weakens with the square of the distance from the Earth's center.

Check

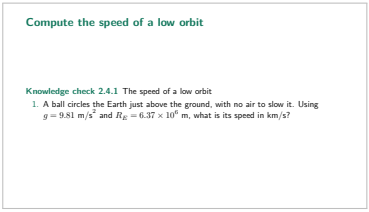
Compute the speed of a low orbit

Knowledge check 2.4.1 The speed of a low orbit

1. A ball circles the Earth just above the ground, with no air to slow it. Using $g = 9.81$ m/s^2 and $R_E = 6.37 \times 10^6$ m, what is its speed in km/s?

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Compute the speed of a low orbit



Answer: 7.9 (within ± 0.1). $v_1 = \sqrt{g\,R_E} = \sqrt{9.81\ \text{m/s}^2 \times 6.37 \times 10^6\ \text{m}} \approx 7.9 \times 10^3$ m/s, which is 7.9 km/s.

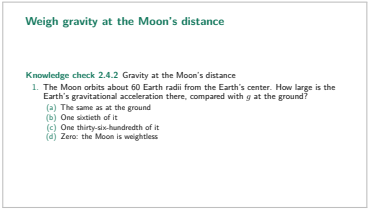
Weigh gravity at the Moon's distance

Knowledge check 2.4.2 Gravity at the Moon's distance

1. The Moon orbits about 60 Earth radii from the Earth's center. How large is the Earth's gravitational acceleration there, compared with g at the ground?
   (a) The same as at the ground
   (b) One sixtieth of it
   (c) One thirty-six-hundredth of it
   (d) Zero: the Moon is weightless

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Weigh gravity at the Moon's distance



Answer: (c) One thirty-six-hundredth of it. Gravity weakens with the square of the distance from the Earth's center, so at $60\,R_E$ it is $g/60^2 = g/3600$, about 2.7×10^{-3} m/s^2. Newton found the same value from the Moon's distance and its period.

What you can now do

- Explain an orbit as a horizontal launch so fast that the ground curves away as fast as the projectile falls, and compute the speed and the period of a low orbit.

References

[1] Newton, A Treatise of the System of the World (1728).

[2] Ling et al., University Physics Volume 1 (2016). ch. 13, “Satellite Orbits and Energy”. https://openstax.org/details/books/university-physics-volume-1

[3] Ling et al., University Physics Volume 1 (2016). ch. 13, “Gravitational Potential Energy and Total Energy”. https://openstax.org/details/books/university-physics-volume-1

Notation

v_0 **Launch speed**. The object's speed at the start of the scenario being analyzed — its initial velocity in a straight-line problem, or its launch speed the instant a projectile leaves the ground. *Units:* meters per second.

t **Time**. Elapsed time since the object was launched or released, measured in seconds. *Units:* seconds after launch.

g **Gravitational acceleration**. The magnitude of gravitational acceleration, treated as constant near the ground. *Units:* meters per second squared.

v_1 **Circular orbital speed**. The horizontal launch speed at which a projectile falls around the Earth on a circle and never reaches the ground. *Units:* meters per second.

R_E **Earth's radius**. The distance from the Earth's center to its surface. *Units:* meters.

Notation (continued)

T **Orbital period**. The time a body on a closed orbit takes to come back to where it started. *Units:* seconds.

v_2 **Escape speed**. The launch speed from which a projectile leaves the Earth and never comes back. *Units:* meters per second.

$\mathbf{a}$ **Acceleration**. The rate at which velocity changes with time. In this course it is held constant over a scenario, so it is the same vector at every instant. *Units:* meters per second squared. $\mathbf{a} = \dfrac{\mathrm{d}\,\mathbf{v}}{\mathrm{d}t}$

r **Orbit radius**. The distance from the Earth's center to a body that orbits it on a circle. *Units:* meters.