Chapter 1.2

Velocity Under Constant Acceleration

From *Explico Playground — Motion*, with changes after version 1.0.0, not yet released. Interactive version: https://danieltuzes.github.io/explico/playground/motion/constant-acceleration/

The time origin

In every scenario of this course, Time is zero at the start of the scenario. Time only increases during a scenario. An object can reverse its direction of motion, but time does not reverse.

1.2.1 Velocity over time

The Velocity of an object states its speed and its direction of motion at one instant. In this chapter, the object moves along one straight line. So the velocity is a signed number: its absolute value is the speed, and its sign is the direction. The Acceleration is constant during the scenario. On one axis, the vector equation (1.1.6) gives the velocity at time t as the Launch speed plus the acceleration multiplied by t:

$$v(t) = v_0 + at \tag{1.2.1}$$

(1.2.1) is exact, not an approximation. The acceleration is defined as the rate of change of the velocity. If the acceleration is constant, integrating this definition over time gives (1.2.1) without an error term [1]. The same integration for a vector is in § 1.1.4.1.

Example 1.2.1: Speeding up and braking at a constant acceleration

Entering a highway

A car enters a straight on-ramp (the curved roadway that lets vehicles merge onto a highway from a surface street) at a Launch speed of 5 m/s and has a constant Acceleration of 2 m/s^2. Substituting these values and a time of 4 s into (1.2.1) gives its velocity:

$$\begin{aligned} v(4) &= 5\ \text{m/s} + 2\ \text{m/s}^2 \times 4\ \text{s} \\ &= 5\ \text{m/s} + 8\ \text{m/s} \\ &= 13\ \text{m/s} \end{aligned}$$

Braking to a stop

A cyclist moves at a Launch speed of 8 m/s and brakes (an acceleration opposite to the velocity decreases the speed) with a constant Acceleration of $-2\ \text{m/s}^2$. Substituting these values and a time of 3 s into (1.2.1) gives its velocity:

$$\begin{aligned} v(3) &= 8\ \text{m/s} + \left(-2\ \text{m/s}^2\right) \times 3\ \text{s} \\ &= 8\ \text{m/s} - 6\ \text{m/s} \\ &= 2\ \text{m/s} \end{aligned}$$

Each second of braking takes 2 m/s off the velocity:

Time	Velocity
0 s	8 m/s
1 s	6 m/s
2 s	4 m/s
3 s	2 m/s

Point at a symbol, in the text or in an equation, to open its explanation. Click the symbol to keep the explanation open while you read.

Table 1.2.1 **The two worked examples at a glance.** Each scenario's launch speed, acceleration and time, and the velocity they give.

Scenario	Launch speed (m/s)	Acceleration (m/s^2)	Time (s)	Velocity (m/s)
Entering a highway	5	2	4	13
Braking to a stop	8	-2	3	2

Table 1.2.1 compares the inputs and the results of the two examples in Example 1.2.1. Knowledge check 1.2.1 tests the same calculation.

Knowledge check 1.2.1: Velocity under constant acceleration

1. A car has an initial velocity of 5 m/s and a constant acceleration of $2\ \text{m/s}^2$. What is its velocity, in m/s, after 4 s?

Answers: end of the chapter.

References

[1] Ling et al., University Physics Volume 1 (2016). ch. 3, "Instantaneous Velocity and Speed". https://openstax.org/details/books/university-physics-volume-1

Notation

x **Horizontal coordinate**. The signed distance from the origin along the horizontal axis, measured positive to the right. *Units:* meters.

y **Vertical coordinate**. The signed distance from the origin along the vertical axis, measured positive upward. *Units:* meters.

$\mathbf{a}$ **Acceleration**. The rate at which velocity changes with time. In this course it is held constant over a scenario, so it is the same vector at every instant. *Units:* meters per second squared.

$$\mathbf{a} = \frac{\mathrm{d}\,\mathbf{v}}{\mathrm{d}t}$$

v_0 **Launch speed**. The object's speed at the start of the scenario being analyzed — its initial velocity in a straight-line problem, or its launch speed the instant a projectile leaves the ground. *Units:* meters per second.

$\mathbf{r}$ **Position**. The object's location at elapsed time t, taken as a vector from a fixed origin so a two-dimensional motion keeps its horizontal and vertical parts separate. *Units:* meters.

$$\mathbf{r} = (x,\ y)$$

t **Time**. Elapsed time since the object was launched or released, measured in seconds. *Units:* seconds after launch.

$v(t)$ **Velocity**. The rate at which position changes with time. It is a vector; in a straight-line problem only its size and sign matter, and that is what the scalar v(t) tracks. *Units:* meters per second.

$$\mathbf{v} = \frac{\mathrm{d}\,\mathbf{r}}{\mathrm{d}t}; \quad v(t) = v_0 + a\,t$$

Answers to the knowledge checks

Knowledge check 1.2.1: Velocity under constant acceleration

1. 13 (accepted within ± 0.1). $v(t) = v_0 + a\,t = 5\ \mathrm{m/s} + 2\ \mathrm{m/s^2} \times 4\ \mathrm{s} = 5\ \mathrm{m/s} + 8\ \mathrm{m/s} = 13\ \mathrm{m/s}$.