Chapter 3.1

Discount curve and forward discounting

From *Credit Products Playground*, with changes after version 0.4.0, not yet released. Interactive version: https://danieltuzes.github.io/credit-and-derivatives/rates/discount-curve-and-forward-discounting/

3.1.1 What you will be able to do

After this lesson, you should be able to read a deterministic discount curve, state its interpolation boundary, and calculate a discount factor between two future dates.

3.1.2 A curve is a dated collection, not one yield

In the earlier lessons, the discount-factor $D(0,t)$ gave the value at valuation-time of a deterministic payment at one payment-time. A discount curve gives the discount factor for many payment times. Each node of the curve is one payment time with its discount factor. Every node must have the same currency, the same valuation date, and the same collateral and funding basis. Nodes with different bases do not form one consistent curve.

The curve in this lesson accepts only positive discount factors at its nodes, and it sets $D(0,0) = 1$. Between two nodes, the logarithm of the discount factor is interpolated linearly. The curve is not extrapolated beyond its final node. Log-linear interpolation is a convention of this lesson. The lesson does not claim that it is the only correct interpolation.

A discount factor can be larger than one. For example, $D(0,1) = 1.01$ implies a negative continuously compounded one-year zero rate. The curve requires positive discount factors, but it does not require discount factors below one.

3.1.3 Discount between future times

Let the local forward-discount-start-time t be earlier than the local forward-discount-end-time T. The forward-discount-factor is defined as the ratio of the two discount factors from valuation time:

$$Z(t,T) = \frac{D(0,T)}{D(0,t)} \tag{3.1.1}$$

Its unit is currency at time t per unit of currency at time T. If $D(0,1) = 0.98$ and $D(0,3) = 0.94$, the forward discount factor from year 1 to year 3 is:

$$Z(1,3) = \frac{0.94}{0.98} = 0.9591836735 \tag{3.1.2}$$

The forward discount factor has its own symbol, Z. The symbol D keeps its meaning: a discount factor from valuation time.

The ratio in (3.1.1) is needed for multi-period valuation. Each lattice node needs the discount factor over its next period. A discount factor from time zero is not the correct discount factor for that period.

3.1.4 Scope boundary

The curve is an input. It is deterministic and not calibrated. Later lessons with state-dependent discounting give a one-period discount factor at each node. They do not assume that today's forward discount factor is realized in every future state.

Knowledge check 3.1.1: Discount curve and forward discounting

1. What does a deterministic curve node D(0,3) = 0.94 mean?
   (a) At time zero, 0.94 currency units buy one currency unit paid at model-year 3
   (b) At model-year 3, 0.94 currency units are paid at time zero
   (c) The annual rate is 94% under every compounding convention
2. Can a valid input curve contain D(0,1) = 1.01 in this lesson?
   (a) Yes; a positive factor above one corresponds to a negative continuously compounded zero rate
   (b) No; every discount factor is mathematically restricted below one
   (c) Only if issuer default is certain
3. Given D(0,1) = 0.98 and D(0,3) = 0.94, calculate the forward discount factor Z(1,3).
4. Given D(0,2) = 0.95 and D(0,5) = 0.88, calculate the forward discount factor Z(2,5).

Answers: end of the chapter.

3.1.5 Sources

Discount-factor pricing and the law-of-one-price interpretation follow Tuckman and Serrat, Chapter 1 §§1.2–1.4 and Chapter 2 §§2.1 and 2.4 [1]. The later derivatives use of dated factors is consistent with Hull's interest-rate and forward-pricing chapters [2].

References

[1] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022).

[2] Hull, Options, Futures, and Other Derivatives (8th ed., 2012).

Notation

T **Forward discount end time**. Later model time at which one currency unit is paid. *Units:* model-years from valuation time.

t **Forward discount start time**. Earlier future model time from which the later payment is valued. *Units:* model-years from valuation time.

$A(0,t)$ **Accumulation factor**. Grows one current unit over a stated future horizon under the selected rate model. *Units:* dimensionless currency-units per current currency-unit.

m **Compounding frequency**. Number of equal compounding periods per year under the stated rate convention, a positive integer fixed by the model convention. *Units:* compounding periods per year.

$D(0,t)$ **Discount factor**. Converts one deterministic future unit into value at valuation time. *Units:* current currency-units per future currency-unit.

$Z(t,T)$ **Forward discount factor**. Value at future model time t of one currency unit paid at later model time T under the stated deterministic curve, with t strictly before T. *Units:* currency at current time per currency-unit at future time.

$j^{(m)}$ **Nominal annual rate**. Annualized rate quote that must be paired with its compounding frequency. *Units:* decimal rate per year.

t_k **Payment time**. Time from the valuation date to one scheduled cash-flow event; a time coordinate, not a calendar date or payment index. *Units:* years from the valuation date.

r_m **Periodic rate**. Rate applied once in each compounding period under the stated convention, derived from the stated nominal annual quote in this model. *Units:* decimal per compounding period.

0 **Valuation time**. Common origin from which later model times and present values are measured. *Units:* years from the valuation date.

Answers to the knowledge checks

Knowledge check 3.1.1: Discount curve and forward discounting

1. **(a)** At time zero, 0.94 currency units buy one currency unit paid at model-year 3. A discount factor is a dated price ratio. It is not itself an annualized rate quotation.

2. **(a)** Yes; a positive factor above one corresponds to a negative continuously compounded zero rate. The domain requires positive factors, not factors below one. Negative rates can produce factors above one.

3. 0.9591836734693877 (accepted within $\pm 1e-9$). Z(1,3) = D(0,3) / D(0,1) = 0.94 / 0.98.

4. 0.9263157894736842 (accepted within $\pm 1e-9$). Z(2,5) = 0.88 / 0.95. The result is time-2 currency per one unit paid at time 5.