# CDS

Part 7 of Credit Products Playground

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Content with changes after version 0.4.0, not yet released
https://danieltuzes.github.io/credit-and-derivatives/

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# Part 7

# CDS

Part 6 gave survival probabilities from a hazard rate, recovery of par, and the present value of a payment that depends on default. A credit default swap exchanges payments that depend on the same default time, so each of its legs is a discounted expectation over the default intervals. The credit-curve chapter also uses the discount curve of Part 3.

This part values a credit default swap in a toy model and connects the value to the way the market quotes it. The first chapter builds the premium and protection legs from exact default-time expectations and solves the par spread. The second chapter converts a market-standard quote into a signed upfront amount against a fixed running coupon, and back. The third chapter separates what the market shows from what the model assumes or solves, and fits a credit curve to several tenors. The last chapter measures a position's sensitivity to the liquid contract's quote and sizes the equivalent notional that offsets it.

## The argument

1. Each leg is a discounted expectation over the default intervals
2. The par spread makes the two legs equal with zero upfront
3. A market quote converts to an upfront amount against a fixed coupon
4. Several tenor marks fit a piecewise-constant hazard curve
5. Bump and reprice gives the quote risk, and the equivalent notional offsets it

## After this part you can

- Interpret a CDS as running-premium payments exchanged for contingent protection, partition default into disjoint time events, and read the protection buyer's signed value as protection minus premium.
- Calculate the positive premium-leg present-value magnitude from scheduled survival-contingent premiums and exact accrued premium integrated over modeled default time under the stated constant-hazard model.
- Calculate the positive protection-leg present-value magnitude by integrating discounted loss-given-default over exact modeled default time under the stated constant risk-neutral hazard.
- Calculate the zero-upfront CDS running spread that equates the exact default-time premium- and protection-leg present-value magnitudes under one internally consistent model.
- Distinguish the lesson's market-standard quote from the fixed running coupon and explain the quote's role in a simplified CDS conversion.
- Calculate the signed upfront amount that makes protection-buyer inception value zero for an input quote-implied premium annuity and fixed running coupon.
- Calculate the market-standard quote implied by a signed upfront amount under the lesson's simplified flat-hazard conversion.
- Classify each quantity in a standard CDS valuation as market-observed, contractual, conventional, or model-solved, and explain why the traded upfront alone identifies the pricing-model hazard rate.

- Interpret a credit curve as the term structure of standard-contract marks at quoted tenors together with the survival curve fitted to them, and distinguish a tenor-by-tenor change of quotation from a fit of one hazard term structure.
- Calculate the survival probability implied by fitted piecewise-constant hazard segments and verify that a tenor's standard contract reprices to its mark under the fitted curve.
- Calculate a CDS position's signed value change for a one-basis-point rise in the liquid tenor's market-standard quote by re-solving the flat hazard rate and revaluing the position for its own side.
- Calculate the notional and side of the most liquid standard CDS that offsets a position's sensitivity to the liquid tenor's quote, and express it as an equivalent ratio to the position notional.

## Builds on

- Foundations
- Rates and curves
- Credit risk

# Chapter 7.1

# CDS premium and protection legs

Interactive version: https://danieltuzes.github.io/credit-and-derivatives/cds/premium-protection-legs-and-pa

## 7.1.1 What a CDS is

A credit default swap (CDS) is a bilateral derivative on a named reference entity. The protection buyer pays a running premium to the protection seller. If a covered credit event occurs before maturity, the seller makes a loss-related protection payment. Unlike buying the entity's bond, entering a CDS does not itself lend principal to the reference entity. Tuckman and Serrat describe the fee leg and the contingent leg (in this course, the premium leg and the protection leg) in Chapter 14[1]; Hull describes the same cash-flow directions for buyer and seller, and the quarterly premium convention[2].

This lesson reports both leg present values as positive magnitudes. From the protection buyer's perspective, the net present value is the protection-leg present value minus the premium-leg present value:

$$PV_0^{\text{buyer}} = PV_0^{\text{prot}} - PV_0^{\text{prem}} \tag{7.1.1}$$

The seller has the opposite signed value in this two-party model.

## 7.1.2 Start from disjoint events

Let the random default-time be $\tau$. Use the local cds-period-index $i$ across the number-of-cds-periods $n$. The schedule has dates $0 = t_0 < t_1 < \cdots < t_n = T$. Define the disjoint interval events

$$G_i = \{t_{i-1} < \tau \le t_i\}, \qquad i = 1, \dots, n \tag{7.1.2}$$

and add $G_0 = \{\tau > T\}$ for survival through maturity. These events are disjoint and exhaustive under the one-default assumption.

For a local cds-event-payoff $Y$ that is constant at $y_i$ on each event, the risk-neutral expectation is:

$$\mathbb{E}^{\mathbb{Q}}[Y] = \sum_{i=0}^{n} y_i \, \mathbb{Q}(G_i) \tag{7.1.3}$$

This is the expectation from disjoint events of the probability lesson, with risk-neutral pricing weights. If $Y$ is not constant within an interval, the expectation uses the conditional means:

$$\mathbb{E}^{\mathbb{Q}}[Y] = \sum_{i=0}^{n} \mathbb{Q}(G_i)\, \mathbb{E}^{\mathbb{Q}}[Y \mid G_i] \tag{7.1.4}$$

For this reason, one payment at the end of the period is not an exact treatment of a default in the middle of the period. If every interval is divided into smaller default-time intervals, the limit of the sum is an integral. Shreve gives the finite disjoint-event expectation rule used here[3]. Tuckman and Serrat's CDS table also weights cash flows by survival and default events[4].

### 7.1.3 The hazard rate gives the probabilities in the formulas

Let $\lambda(u)$ be the risk-neutral default intensity at time $u$, given survival to $u$. It is a deterministic function of time, measured per model-year under the risk-neutral-probability-measure $\mathbb{Q}$. It generalizes the constant Constant hazard rate $\lambda$ of this lesson. Its meaning is the conditional intensity of the credit lesson, but it need not be constant: $\lambda(u)$ is the proportional rate at which the survival probability decreases at $u$, given survival to $u$. As a differential equation, this meaning is:

$$-\,dS^{\mathbb{Q}}(0,u) = \lambda(u)\, S^{\mathbb{Q}}(0,u)\, du, \qquad S^{\mathbb{Q}}(0,0) = 1 \tag{7.1.5}$$

Dividing by the (positive) survival probability turns the left side into a derivative of its logarithm, $-d\ln\left(S^{\mathbb{Q}}(0,u)\right) = \lambda(u)\, du$. Integrating from 0 to $t$ and using $\ln\left(S^{\mathbb{Q}}(0,0)\right) = \ln(1) = 0$ gives the survival probability:

$$S^{\mathbb{Q}}(0,t) = \exp\left(-\int_0^t \lambda(u)\; du\right) \tag{7.1.6}$$

Integrating the same differential equation over one period, not from 0, gives the interval default probability for a general hazard rate:

$$\Delta q_i^{\mathbb{Q}} = \int_{t_{i-1}}^{t_i} \lambda(u)\, S^{\mathbb{Q}}(0,u)\, du = S^{\mathbb{Q}}(0,t_{i-1}) - S^{\mathbb{Q}}(0,t_i) \tag{7.1.7}$$

The constant Constant hazard rate $\lambda$ of this lesson is the special case $\lambda(u) \equiv \lambda$. Then $\int_0^t \lambda(u)\, du = \lambda t$, and the two results above become:

$$S^{\mathbb{Q}}(0,t) = \exp(-\lambda t), \qquad \Delta q_i^{\mathbb{Q}} = \exp(-\lambda t_{i-1}) - \exp(-\lambda t_i) \tag{7.1.8}$$

The next two sections, on the premium leg and on the protection leg, use the general $\lambda(u)$. The section on closed forms after them uses the constant $\lambda$.

These probabilities are pricing probabilities, not forecasts, for a constant and for a general hazard rate. A hazard rate implied from CDS prices can be a risk-neutral quantity rather than a prediction of realized defaults[5]. The constant-hazard survival relation is given in §14.6 and Appendix A14.1[6].

### 7.1.4 Default in the middle of a period

Suppose default occurs at $u$ in $(t_{i-1}, t_i]$. The input recovery-rate is $R$.

- The scheduled premium at $t_i$ is not paid, because the reference entity did not survive to that date.

- The buyer pays premium accrued from $t_{i-1}$ to $u$. In this model its year fraction is $a_i(u) = u - t_{i-1}$.
- The seller pays protection of $\text{LGD}N = (1 - R)N$.
- Both amounts triggered by default are discounted from the modeled default time $u$, not from $t_i$.

Hull notes that a final accrued premium is paid when default occurs between scheduled dates[2]. The half-period and midpoint calculations in Hull and in Tuckman and Serrat are discrete approximations[7][8]. This lesson integrates over the modeled default time exactly, for a general hazard rate and a general discount curve. The closed forms later in the lesson use a constant hazard rate and a constant interest rate.

### 7.1.5 Exact premium-leg expectation in this model

Let $D(0, u)$ be the general dated discount curve of the rates lesson; this section does not assume a flat rate. The cds-scheduled-premium-annuity is:

$$A_0^{\text{sched}} = \sum_{i=1}^{n} \alpha_i D(0, t_i) S^{\mathbb{Q}}(0, t_i) \tag{7.1.9}$$

The exact cds-accrued-premium-annuity is:

$$A_0^{\text{accr}} = \sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} a_i(u) D(0, u) \lambda(u) S^{\mathbb{Q}}(0, u) \, \mathrm{d}u \tag{7.1.10}$$

The cds-premium-annuity is the sum of the two annuities. The premium-leg present value is the annual spread, as a decimal, multiplied by the notional and by the premium annuity:

$$A_0^{\text{prem}} = A_0^{\text{sched}} + A_0^{\text{accr}}, \qquad PV_0^{\text{prem}} = sNA_0^{\text{prem}} \tag{7.1.11}$$

A spread quoted in basis-point per year is divided by 10,000 before it is used in (7.1.11).

### 7.1.6 Exact protection-leg expectation in this model

The protection-leg present value, as a positive magnitude, is:

$$PV_0^{\text{prot}} = N\text{LGD} \sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} D(0, u) \lambda(u) S^{\mathbb{Q}}(0, u) \, \mathrm{d}u \tag{7.1.12}$$

Equation (7.1.12) is an expectation. Each possible payoff at a default time is discounted and multiplied by the risk-neutral probability density of that default time. The integral then adds over all default times.

### 7.1.7 Closed forms and the zero-upfront par spread

This section uses a constant Constant hazard rate, $\lambda(u) \equiv \lambda$, which is a special case of the cds-general-hazard-rate. It also uses a constant continuously-compounded-risk-free-rate $r$. So the general discount curve of the last two sections is $D(0,u) = \exp(-ru)$.

For the regular model-year schedule, $\alpha_i = t_i - t_{i-1}$ and the maturity-time is $T = t_n$. Define the abbreviation $\kappa = r + \lambda$. Then the scheduled-premium annuity and the accrued-premium annuity are:

$$A_0^{\text{sched}} = \sum_{i=1}^{n} \alpha_i \exp(-\kappa t_i) \tag{7.1.13}$$

$$A_0^{\text{accr}} = \sum_{i=1}^{n} \lambda \exp(-\kappa t_{i-1}) \frac{1 - \exp(-\kappa\alpha_i)(1 + \kappa\alpha_i)}{\kappa^2} \tag{7.1.14}$$

The cds-protection-factor, per unit of notional, is:

$$B_0^{\text{prot}} = \text{LGD}\, \lambda \frac{1 - \exp(-\kappa T)}{\kappa} \tag{7.1.15}$$

At $\kappa = 0$, the continuous limits replace the two fractions by ${\alpha_i}^2/2$ and $T$, respectively. The implementation uses numerically stable forms of those limits.

The cds-par-spread $s^\star$ is the running spread that makes the legs equal with no upfront amount:

$$s^\star = \frac{B_0^{\text{prot}}}{A_0^{\text{prem}}}, \qquad PV_0^{\text{prem}}(s^\star) = PV_0^{\text{prot}} \tag{7.1.16}$$

Tuckman and Serrat state the condition that the fair spread makes the two legs equal, and the relationship between spread and upfront[9]. The next lesson distinguishes this model par spread from a standard running coupon and a signed upfront amount.

**Example 7.1.1: Exact default-time CDS legs**

**A finite event partition**

Suppose a two-period protection payoff, simplified to one value per event, is USD 6,000 on $G_1$, USD 5,800 on $G_2$, and zero on the survival event $G_0$. The risk-neutral probabilities of $G_1$, $G_2$, and $G_0$ are 0.02, 0.03, and 0.95. The expected payoff before discounting is:

$$0(0.95) + 6{,}000(0.02) + 5{,}800(0.03) = \text{USD } 294$$

This finite sum is exact only because the payoff is constant on each event. In an exact CDS valuation, the discounted payoff varies with the default time inside an event. So the exact valuation uses an integral over all default times.

**One-year exact legs**

Take notional $N = \text{USD } 1{,}000{,}000$, running spread $s = 0.01$ per year (100 bp/year), recovery $R = 0.40$, constant risk-neutral hazard $\lambda = 0.02$ per model-year, continuously compounded risk-free rate $r = 0.04$ per model-year, two equal periods, and maturity $T = 1$ model-year.
The tested domain code gives:

| Factor | Value |
|---|---|
| Scheduled premium annuity | 0.956105034 model-years |
| Accrued-premium annuity | 0.004828691 model-years |
| Premium annuity | 0.960933725 model-years |
| Protection factor | 0.011647093 per unit notional |

The premium-leg present value is:

$$PV_0^{\text{prem}} = 0.01(1{,}000{,}000)(0.960933725) \approx \text{USD } 9{,}609.34$$

The protection-leg present value is:

$$PV_0^{\text{prot}} = 1{,}000{,}000(0.011647093) \approx \text{USD } 11{,}647.09$$

The present value of the accrued premium is USD 48.29. It is small in this example. The model calculates it at every possible default time; it does not approximate it by one half of a period.

**Solve the exact par spread**

With the same inputs, the par spread is:

$$s^\star = \frac{0.011647093}{0.960933725} \approx 0.012120600 = 121.206 \text{ bp/year}$$

At the unrounded par spread, the present value of each leg is approximately USD 11,647.09. The notional cancels in the par spread, because both legs are proportional to the same notional.

### 7.1.8 Generalizing the premium leg: an extinguishing swap

This lesson uses the name **extinguishing swap** for the construction below. The name belongs to this course; it is not a standard market term or textbook term. An extinguishing swap is any schedule of cash flows that stops permanently at the default of the reference entity. The premium leg above has this property.

Consider the same dates $0 = t_0 < t_1 < \cdots < t_n = T$ and the same reference entity. Replace the scheduled premium $sN\alpha_i$ at each date with an arbitrary deterministic, signed net cash flow $c_i$ for one named perspective. Under the signed cash-flow convention of this course, a positive $c_i$ is received and a negative $c_i$ is paid. As with the premium leg, a scheduled amount is paid only on survival to its date. Every later scheduled amount is extinguished at the default time. In this lesson, $c_i$ is an input and deterministic; it does not depend on a floating rate. If a cash flow accrues continuously between dates, an accrued-at-default term can be added with the same construction as the cds-accrued-premium-annuity. This lesson does not repeat that term.

The expectation from disjoint events, used for both CDS legs above, gives the cds-extinguishing-swap-present-value of the scheduled cash flows. It is the same survival-weighted discounting that gave $A_0^{\text{sched}}$:

$$PV_0^{\text{ext}} = \sum_{i=1}^{n} c_i \, D(0,t_i) \, S^{\mathbb{Q}}(0,t_i) \tag{7.1.17}$$

With $c_i = sN\alpha_i$ for every $i$, the present value is $sNA_0^{\text{sched}}$, the scheduled-payment term of the premium leg. So the premium leg is the special case of an extinguishing swap whose cash flows are one running spread paid on a fixed notional.

**Example 7.1.2: A two-date extinguishing swap**

**A two-date extinguishing swap**

Use the model of the "One-year exact legs" example: $\kappa = r + \lambda = 0.06$ per model-year, with semiannual dates $t_1 = 0.5$ and $t_2 = 1$. The discounted-survival factors are:

$$D(0,t_1)S(0,t_1) = \exp(-0.06\times 0.5) \approx 0.970445534, \qquad D(0,t_2)S(0,t_2) = \exp(-0.06\times 1) \approx 0.941764534$$

Suppose this extinguishing swap's schedule is a USD 30,000 receipt at $t_1$ and a USD 12,000 payment at $t_2$, so $c_1 =$ USD 30,000 and $c_2 = -$USD 12,000. The present value of the extinguishing swap is:

$$PV_0^{\text{ext}} = 30{,}000(0.970445534) + (-12{,}000)(0.941764534) \approx \text{USD } 17{,}812.19$$

Both terms use the same $D(0,t_i)S(0,t_i)$ factors as the premium leg. Only the signed cash flows $c_i$ are different from the premium leg's $sN\alpha_i$.

### 7.1.9 Why there is no volatility input here

Risk-neutral valuation does not remove volatility. A change from the real-world measure to $\mathbb{Q}$ changes the pricing probabilities, and in common diffusion models it changes the drift. It does not change the size of the random fluctuations. Shreve shows that the Brownian volatility term remains after the risk-neutral change of measure[10].

This CDS model needs no separate volatility parameter for two reasons. First, its risk-free rate, hazard rate, and recovery rate are deterministic. Second, both legs are expectations that are linear in the cash flows. In other cases, the distributions, the dependence, and the volatility of the inputs could matter: if rates, the hazard rate, or recovery were stochastic, or if the product were a CDS option, had wrong-way risk, or had nonlinear exposure. The risk-neutral measure gives the pricing weights. The restrictive model assumptions, not the measure, remove the need for a separate volatility input.

### 7.1.10 Both legs for one contract

Figure 7.1.1 shows the present value of each leg for one contract in this model. The explorer integrates the accrued premium and the protection payment over the exact modeled default time.

### 7.1.11 Check your understanding

The assessment provides direct and transfer evidence for leg perspective, premium-leg magnitude, protection-leg magnitude, and par spread.

**Knowledge check 7.1.1: CDS legs and par spread**

1. Under the lesson sign convention, both CDS legs are reported as positive present-value magnitudes. Which expression gives the protection buyer's signed net value?

   (a) Protection-leg magnitude minus premium-leg magnitude

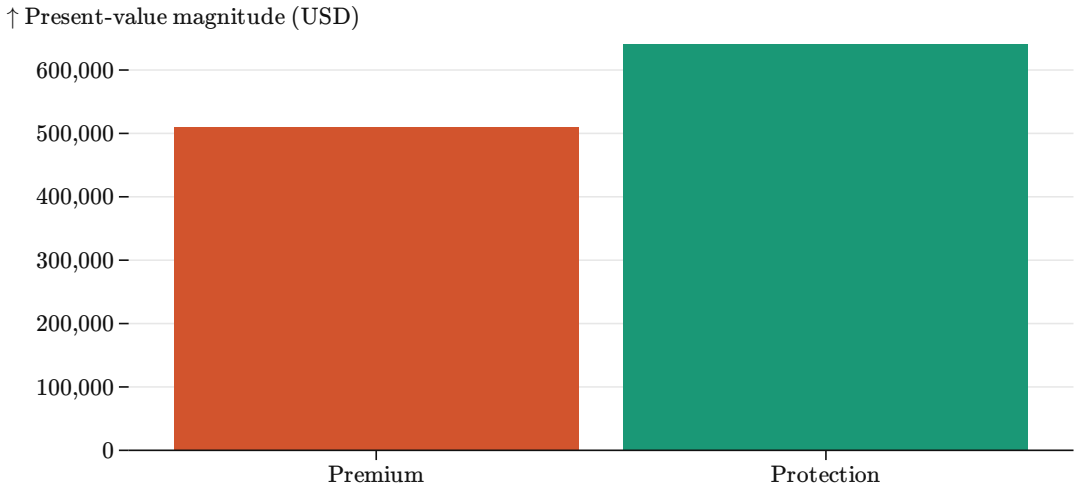



Figure 7.1.1 **CDS leg explorer.** The present values of the premium leg and the protection leg, as positive magnitudes in USD, for a notional of USD 10,000,000 and four equal model periods a year. The protection buyer's value is the protection leg minus the premium leg. Shown for a contractual spread of 120.0 bp/year, a constant risk-neutral hazard rate of 2.5%/year, a recovery rate of 40%, a continuously compounded risk-free rate of 4% and a maturity of 5 years: premium leg USD 509,703, protection leg USD 640,321.

(b) Premium-leg magnitude minus protection-leg magnitude

(c) Protection-leg magnitude plus premium-leg magnitude

2. A CDS has a positive protection-leg present-value magnitude of USD 35 and a positive premium-leg present-value magnitude of USD 28. What is the protection buyer's signed net value in USD?

   (a) +7

   (b) -7

   (c) +63

3. In the lesson's exact default-time model, a one-year annual-pay CDS has scheduled-premium annuity 0.941764533584 model-years and exact accrued-on-default annuity 0.009608857782 model-years. Calculate the positive premium-leg present-value magnitude in USD for notional USD 1,000,000 and running spread 100 basis points (0.01 per year).

4. A two-year quarterly CDS has notional USD 3,000,000 and running spread 125 basis points (0.0125 per year). Exact integration under the input constant-hazard model gives scheduled-premium annuity 1.829782217887 model-years and accrued-on-default annuity 0.011512762522 model-years. Calculate the positive premium-leg magnitude in USD.

5. For a one-year CDS, exact integration over modeled default time gives a discounted risk-neutral default-density integral of 0.019411822139. Calculate the positive protection-leg magnitude in USD for notional USD 1,000,000 and recovery rate 0.40.

6. Use the exact flat-hazard protection formula $N(1-R)\lambda[1-\exp(-(r+\lambda)T)]/(r+\lambda)$. Calculate the positive protection-leg magnitude in USD for $N$ = USD 3,000,000, $R = 0.35$, $\lambda = 0.05$ per year, $r = 0.03$ per year, and $T = 2$ years.

7. Calculate the zero-upfront par spread in basis points per year when the exact premium annuity is 0.951373391366 model-years and the exact protection factor is 0.011647093283 per unit notional.

8. A different exact default-time model produces scheduled-premium annuity 1.829782217887 model-years, accrued-on-default annuity 0.011512762522 model-years, and protection factor 0.060066585732 per unit notional. Calculate its zero-upfront par spread in basis points per year.

*Answers:* *end of the chapter.*

### 7.1.12 Model boundary and review note

The valuation is exact only inside this simplified model. The premium leg and the protection leg are derived for a general, possibly time-varying risk-neutral hazard rate and a general dated discount curve. The closed forms, the numeric examples, the explorer, and the assessment use a special case: one constant risk-neutral hazard rate, one deterministic continuously compounded risk-free rate, a deterministic recovery rate, and settlement at the default time. The extinguishing swap is a construction of this lesson that generalizes the premium leg; it is not a named market instrument. The model omits calendar schedules, day counts, business-day rules, payment delays, term-structure calibration, auctions, deliverables, credit-event legal definitions, counterparty risk, collateral, funding, restructuring, and transaction costs. All materially changed content remains ‘draft‘ pending independent human review.

## References

[1] Tuckman & Serrat, Fixed Income Securities: Tools for Today’s Markets (4th ed., 2022). §14.5, printed pp. 361–362.

[2] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). §24.1, printed pp. 548–550.

[3] Shreve, Stochastic Calculus for Finance II: Continuous-Time Models (2004). Ch. 1 §1.3, printed pp. 13–18.

[4] Tuckman & Serrat, Fixed Income Securities: Tools for Today’s Markets (4th ed., 2022). §14.6, printed pp. 368–370, Table 14.10 and eqs. 14.6–14.7.

[5] Tuckman & Serrat, Fixed Income Securities: Tools for Today’s Markets (4th ed., 2022). §14.7, printed p. 371.

[6] Tuckman & Serrat, Fixed Income Securities: Tools for Today’s Markets (4th ed., 2022). printed pp. 367–368 and 505, eqs. 14.4–14.5 and A14.1–A14.2.

[7] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). §24.2, printed pp. 551–554, Tables 24.1–24.4.

[8] Tuckman & Serrat, Fixed Income Securities: Tools for Today’s Markets (4th ed., 2022). §14.6, printed pp. 368–370, Table 14.10.

[9] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). §14.6, printed pp. 370–371, and Appendix A14.2, printed pp. 506–507.

[10] Shreve, Stochastic Calculus for Finance II: Continuous-Time Models (2004). Ch. 5 §5.2.2, printed pp. 214–217.

## Answers to the knowledge checks

**Knowledge check 7.1.1: CDS legs and par spread**

1. **(a)** Protection-leg magnitude minus premium-leg magnitude. The protection leg is an inflow and the premium leg is an outflow for the protection buyer, so signed net value is protection minus premium.

2. **(a)** +7. From the protection buyer's perspective, net value is 35 - 28 = +7.

3. 9513.73391366 (accepted within ±0.01). The exact total premium annuity is 0.941764533584 + 0.009608857782 = 0.951373391366 model-years. The positive premium magnitude is 0.01 × 1,000,000 × 0.951373391366 = USD 9,513.73391366.

4. 69048.56176534 (accepted within ±0.01). The total exact premium annuity is 1.841294980409 model-years. Multiplying 0.0125 × 3,000,000 × 1.841294980409 gives USD 69,048.56176534. The accrued factor comes from integrating default time, not a half-period replacement.

5. 11647.0932834 (accepted within ±0.01). Loss given default is 1 - 0.40 = 0.60. The exact protection magnitude is 1,000,000 × 0.60 × 0.019411822139 = USD 11,647.0932834.

6. 180199.757197 (accepted within ±0.01). Here r + $\lambda$ = 0.08. Substitution gives 3,000,000 × 0.65 × 0.05 × [1 - exp(-0.16)] / 0.08 = USD 180,199.757197. This discounts over actual modeled default time through the integral's closed form.

7. 122.423996602 (accepted within ±0.001). The par annual decimal spread is 0.011647093283 / 0.951373391366 = 0.0122423996602. Multiplying by 10,000 gives 122.423996602 basis points per year; notional cancels.

8. 326.219244453 (accepted within ±0.001). The exact total premium annuity is 1.841294980409. The par spread is 0.060066585732 / 1.841294980409 = 0.0326219244453 per year, or 326.219244453 basis points per year.

# Chapter 7.2

# CDS market-standard quote and upfront

Interactive version: https://danieltuzes.github.io/credit-and-derivatives/cds/market-standard-quote-and-upfr

## 7.2.1 What you will be able to do

After this lesson, you should be able to:

- distinguish a conventional spread quote from the contract's fixed running coupon;
- identify which quantity is an input and which is solved in each quotation direction;
- calculate the signed upfront amount that makes protection-buyer inception value zero;
- reverse the simplified conversion without incorrectly freezing a quote-dependent premium annuity.

## 7.2.2 Three quantities with three different roles

Standardized CDS quotation separates a fixed running coupon from a market-determined upfront amount. Textbook treatments describe how to convert a quoted spread, treated as a par spread, into the value of a contract with a fixed coupon. The official converter specification accepts either the spread or the upfront and returns the other. [1] [2] [3]

This lesson uses the name **market-standard quote** (**MSQ**) for this conventional quoted spread. The name belongs to this course. The books and the official specification more commonly say *quoted spread*, *conventional spread*, or *spread*. This lesson does not claim that "MSQ" is a universal abbreviation.

The three quantities have different roles, but they are not three independent inputs:

- the cds-market-standard-quote $s_{\mathrm{MSQ}}$ is treated as a zero-upfront cds-par-spread and calibrates this lesson's flat Constant hazard rate $\lambda$;
- the cds-standard-coupon $c_{\mathrm{std}}$ determines the running premium cash flows;
- the cds-upfront-amount $U_0$ offsets the difference in value at inception.

The phrase "upfront fee" is also common. This lesson uses **upfront amount**, because the amount is signed. The sign of $U_0$ means:

- $U_0 > 0$: the protection buyer pays;
- $U_0 = 0$: no upfront cash changes hands in the toy model;
- $U_0 < 0$: the protection buyer receives $-U_0$.

In the formulas, rates are decimals per year. Convert a basis-point quote to decimal form by dividing by 10,000.

### 7.2.3 First decide which quantities are inputs and which are solved

The equation that connects the par spread and the hazard rate can be used in two directions. So a quantity can be an input in one calculation and solved in another. Table 7.2.1 shows the inputs and the solved quantities of each calculation.

Table 7.2.1 **Inputs and solved quantities of each calculation.** For the model par spread and for each direction of the quote conversion, the quantities taken as inputs and the quantities solved or calculated.

| Calculation | Input | Solved or calculated |
|---|---|---|
| Model par spread | Hazard rate, recovery, discount input, maturity, and cash-flow conventions | Par spread $s^\star$ |
| MSQ to upfront | Market quote $s_{\mathrm{MSQ}}$, standard coupon $c_{\mathrm{std}}$, and the converter conventions | Flat hazard rate $\lambda$, then signed upfront $U_0$ |
| Upfront to MSQ | Market upfront $U_0$, standard coupon $c_{\mathrm{std}}$, and the converter conventions | Flat hazard rate $\lambda$, then $s_{\mathrm{MSQ}}$ |

If the hazard rate is an input, the par spread is the output:

$$s^\star = \frac{PV_0^{\mathrm{prot}}(\lambda)}{NA_0^{\mathrm{prem}}(\lambda)} \tag{7.2.1}$$

If the MSQ is the input instead, (7.2.1) is a calibration equation: the quoted spread is the input, and the flat hazard rate is the unknown. If the upfront amount is the input, the upfront balance is the calibration equation. The market participant or the price source provides the market level. The standard converter specifies how the spread and the upfront are converted into each other under common assumptions. [4]

So do not fix both $s_{\mathrm{MSQ}}$ and $\lambda$ independently. When the other model inputs are fixed, $s_{\mathrm{MSQ}}$ and $\lambda$ must together satisfy the zero-upfront condition.

### 7.2.4 Why one upfront amount is enough at inception

For fixed model inputs, one positive cds-protection-leg-present-value summarizes all future protection cash flows. One positive cds-premium-leg-present-value summarizes all fixed-coupon cash flows. The difference of the two present values is one currency amount. So one cash transfer at time zero can offset that difference exactly. The upfront balance is:

$$PV_0^{\text{prot}} = PV_0^{\text{prem}} + U_0 \tag{7.2.2}$$

Equivalently, the value to the protection buyer at inception, after the signed upfront amount, is zero:

$$0 = PV_0^{\text{prot}} - PV_0^{\text{prem}} - U_0 \tag{7.2.3}$$

This is a value identity, not a claim that the CDS becomes riskless. After inception its value can change when credit, interest-rate, recovery, liquidity, or other inputs change.

### 7.2.5 Risk-neutral does not mean risk-free

The calibration uses **risk-neutral pricing probabilities**. They are not necessarily real-world default forecasts. Under risk-neutral valuation, probabilities are pricing weights, chosen consistently with traded prices and risk-free discounting. "Risk-free" describes the discount input, not the probability measure. So the hazard rate inferred from prices can differ from an estimated real-world default intensity. [5] [6]

A change of probability measure does not remove volatility. This converter has no separate volatility input, for two reasons. Its discount rate, hazard rate, recovery rate, and schedule are deterministic. Its leg values are expectations that are linear in the one-default cash flows. A model with stochastic rates, stochastic default intensity, stochastic recovery, wrong-way dependence, or CDS options can require volatility and correlation inputs. The risk-neutral measure gives pricing weights; it does not remove those risks.

### 7.2.6 Convert MSQ to signed upfront

The conversion must use one internally consistent model. For each candidate constant hazard rate, calculate the positive cds-premium-annuity $A_0^{\text{prem}}(\lambda)$ and the protection-leg present value. First, solve the zero-upfront quote condition for $\lambda$:

$$s_{\text{MSQ}} N A_0^{\text{prem}}(\lambda) = PV_0^{\text{prot}}(\lambda) \tag{7.2.4}$$

The root $\lambda$ of (7.2.4) is the flat risk-neutral hazard rate implied by the MSQ, under the assumptions of this lesson on the schedule, discounting, recovery, and accrual at default. The premium-leg present value with the fixed coupon is then:

$$PV_0^{\text{prem}}(c_{\text{std}}; \lambda) = c_{\text{std}} N A_0^{\text{prem}}(\lambda) \tag{7.2.5}$$

Substituting (7.2.4) and (7.2.5) into the upfront balance (7.2.2) gives the upfront amount:

$$\begin{aligned} U_0 &= PV_0^{\text{prot}}(\lambda) - c_{\text{std}} N A_0^{\text{prem}}(\lambda) \\ &= N A_0^{\text{prem}}(\lambda) \left( s_{\text{MSQ}} - c_{\text{std}} \right) \end{aligned} \tag{7.2.6}$$

The substitution in (7.2.6) is valid only because every term uses the same model and the same calibrated hazard rate. The premium annuity and the notional are positive. So an MSQ above the fixed coupon gives a positive upfront amount, which the buyer pays. An MSQ below the fixed coupon gives a negative upfront amount, which the buyer receives.

### 7.2.7 Convert signed upfront back to MSQ

When the input is a signed upfront amount, do not rearrange the last line of (7.2.6) with the annuity held fixed. In this flat-hazard converter, both the protection-leg present value and the premium annuity change with the unknown hazard rate. Instead, solve this equation for the hazard rate:

$$U_0 = PV_0^{\text{prot}}(\lambda) - c_{\text{std}} N A_0^{\text{prem}}(\lambda) \tag{7.2.7}$$

Solve (7.2.7) for a non-negative $\lambda$. Then (7.2.8) gives the MSQ from this $\lambda$:

$$s_{\text{MSQ}} = \frac{PV_0^{\text{prot}}(\lambda)}{N A_0^{\text{prem}}(\lambda)} \tag{7.2.8}$$

If a curve and its annuity have already been calibrated and are held fixed, rearranging (7.2.6) gives a shortcut:

$$s_{\text{MSQ}} = c_{\text{std}} + \frac{U_0}{N A_0^{\text{prem}}(\lambda)} \tag{7.2.9}$$

This shortcut is not the full conversion from upfront to quote when the quote itself determines $\lambda$, because then the denominator changes with the quote.

### 7.2.8 When MSQ and par spread are equal

Within one internally consistent risk-neutral model, the MSQ and the par spread are equal. After the converter has inferred one $\lambda$, from the spread or from the upfront amount, the following quantities are equal:

$$\begin{aligned} s_{\text{MSQ}} &= s^\star \\ &= \frac{PV_0^{\text{prot}}(\lambda)}{N A_0^{\text{prem}}(\lambda)} \\ &= c_{\text{std}} + \frac{U_0}{N A_0^{\text{prem}}(\lambda)} \end{aligned} \tag{7.2.10}$$

The equality in (7.2.10) is a risk-neutral pricing identity. No real-world default probability enters it. The equality requires the same pricing probabilities, discount inputs, recovery, schedule, default-accrual treatment, and settlement convention in every term.

The phrase **par spread from another curve** needs more care. The observed trade with a standard coupon gives one combined price:

$$U_0 = PV_0^{\text{prot}} - c_{\text{std}} N A_0^{\text{prem}} \tag{7.2.11}$$

This price does not show the protection-leg present value and the premium annuity separately. The standard converter separates them by assuming its constant-hazard model. A different risk-neutral term structure can reprice the same contract, with its fixed coupon and upfront amount, but with a different premium annuity. That term structure then gives a different par coupon to a hypothetical zero-upfront contract. The official converter itself distinguishes its single constant hazard rate from the term structure of a fuller credit-curve model. [7]

This difference is not an arbitrage by itself, because the observed fixed-coupon CDS and the hypothetical zero-upfront CDS have different premium cash flows. No arbitrage requires identical tradable cash flows to have the same price. But suppose a zero-upfront CDS that pays $s_{\text{MSQ}}$ could

be traded on otherwise identical terms. Then a curve calibrated consistently to that traded contract would have to give it zero value. [8]

**Example 7.2.1: Quote and upfront conversions**

**MSQ to buyer-paid upfront**

Consider a five-year CDS with quarterly premiums and notional $N$ = USD 10,000,000, 2% continuously compounded risk-free rate, 40% recovery, MSQ $s_{\text{MSQ}} = 0.015$ per year (150 basis points per year), and fixed coupon $c_{\text{std}} = 0.010$ per year (100 basis points per year).
Under the flat-hazard model of this lesson, with exact default times, the calibration to the quote gives:

$$\lambda = 0.024937617 \text{ per year}, \qquad A_0^{\text{prem}}(\lambda) = 4.466915290 \text{ model-years}$$

Because the calibrated quote is a par spread, the protection-leg present value is:

$$PV_0^{\text{prot}} = 0.015(10{,}000{,}000)(4.466915290) = \text{USD } 670{,}037.29$$

The premium-leg present value with the fixed coupon is:

$$PV_0^{\text{prem}} = 0.010(10{,}000{,}000)(4.466915290) = \text{USD } 446{,}691.53$$

The upfront amount is the difference of the two present values:

$$U_0 = 670{,}037.29 - 446{,}691.53 = \text{USD } 223{,}345.76$$

The upfront amount is positive, so the protection buyer pays it. As a check, $670{,}037.29 - 446{,}691.53 - 223{,}345.76 = 0$, up to the rounding of the displayed values.

**Why reverse conversion is a solve**

Suppose the input for the same contract is the signed upfront amount $U_0$ = USD 223,345.76, and the MSQ is unknown. A solver changes the flat $\lambda$ and recalculates both the protection-leg present value and the premium annuity, until (7.2.7) holds.
The root is approximately $\lambda = 0.024937617$ per year. Only with this root does (7.2.8) give $s_{\text{MSQ}} \approx 0.015$ (150 basis points per year).
The shortcut with the annuity fixed at 4.466915290 gives the same answer here, but only because this annuity was already calculated at the solution's hazard rate. Before the solve, this annuity is unknown.

### 7.2.9 What a real standard converter adds

The official converter uses dates and market conventions. Its inputs include the trade date and the maturity date, the notional, the standard coupon, the recovery rate, the discount inputs, and either the spread or the upfront. Its outputs include the other quotation and a cash-settlement amount. [3]

This lesson is not an implementation of that standard model. It uses model-year periods and a single flat hazard rate. A production conversion must also handle calendar schedules, day-count and business-day conventions, curve construction and interpolation, settlement timing, the accrued amount, and other contract details. The identities of this lesson explain why the quote, the coupon, and the upfront amount differ. But the numbers in this lesson are not cash-settlement instructions for a trade.

### 7.2.10 Check your understanding

The assessment separates quote interpretation, signed upfront arithmetic, and the reverse flat-hazard solve. Each competency has direct and transfer evidence.

**Knowledge check 7.2.1: CDS market-standard quote and upfront**

1. This lesson uses MSQ as a local abbreviation for market-standard quote. Which statement matches its simplified conversion model?

   (a) MSQ is the conventional quoted spread treated as a zero-upfront par spread for calibrating the flat hazard rate

   (b) MSQ is always the fixed running coupon paid on the contract

   (c) MSQ is the signed upfront cash amount divided by notional

2. A simplified converter is given an MSQ of 180 basis points per year and a standard coupon of 100 basis points per year. Which rate determines the scheduled running premium cash flows?

   (a) The 100-basis-point standard coupon

   (b) The 180-basis-point MSQ

   (c) The 80-basis-point difference

3. Calculate the signed upfront amount in USD when notional is USD 10,000,000, the quote-implied premium annuity is 4.0 model-years, MSQ is 150 basis points per year, and the fixed coupon is 100 basis points per year. Positive means paid by the protection buyer.

4. Calculate the signed upfront amount in USD when notional is USD 5,000,000, the quote-implied premium annuity is 2.5 model-years, MSQ is 300 basis points per year, and the fixed coupon is 500 basis points per year. Positive means paid by the protection buyer; enter a negative number if the buyer receives cash.

5. For a curve that is already calibrated and held fixed, calculate the market-standard quote in basis points per year from notional USD 8,000,000, premium annuity 4.0 model-years, fixed coupon 100 basis points per year, and signed upfront USD 160,000 paid by the protection buyer.

6. Use the lesson's one-year flat-hazard model with one annual period, zero continuously compounded risk-free rate, recovery 40%, notional USD 10,000,000, and fixed coupon 1% per year. For hazard rate lambda, A(lambda) = (1 - exp(-lambda))/lambda and the par quote is (1 - 0.40)lambda. A signed upfront of USD 195,082.302 is paid by the protection buyer. Root solve U0/N = A(lambda)[(1 - 0.40)lambda - 0.01], then report the quote in basis points per year.

*Answers:* *end of the chapter.*

### 7.2.11 Model boundary and review note

This AI-assisted lesson and all cited locators remain `draft` pending human editorial and quantitative review. All input values are invented for teaching. The converter assumes one deterministic discount rate, one deterministic recovery rate, one constant risk-neutral hazard rate, regular model-year periods, exact default-time accrued premium inside that toy schedule, and an upfront payment at time zero. It omits the contract details and numerical methods listed above, as well as stochastic rates, stochastic intensity or recovery, volatility and correlation calibration, counterparty risk, collateral, funding, liquidity, and transaction costs.

## References

[1] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.6, printed pp. 366-371; Appendix A14.2, printed pp. 506-507, eqs. A14.5-A14.7.

[2] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.4, printed pp. 556-557.

[3] International Swaps and Derivatives Association, ISDA CDS Standard Model and Standard CDS Contract Converter Specification. Standard CDS Contract Converter Specification, version May 5, 2009, Functionality and Specification, printed p. 1. https://www.cdsmodel.com/documentation.html

[4] International Swaps and Derivatives Association, ISDA CDS Standard Model and Standard CDS Contract Converter Specification. Standard CDS Contract Converter Specification, version May 5, 2009, Functionality and Specification, printed pp. 1 and 4. https://www.cdsmodel.com/documentation.html

[5] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 7 §7.3, printed pp. 182-184, eqs. 7.7-7.8; Ch. 14 §14.7, printed p. 371.

[6] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 23 §23.5, printed pp. 528-530.

[7] International Swaps and Derivatives Association, ISDA CDS Standard Model and Standard CDS Contract Converter Specification. Standard CDS Examples, version October 26, 2012, printed p. 2 (the converter uses a single flat hazard rate rather than a term structure of flat spreads). https://www.cdsmodel.com/documentation.html

[8] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 1 §§1.3-1.4, printed pp. 53-57 (law of one price, replication, and arbitrage enforcement).

## Answers to the knowledge checks

**Knowledge check 7.2.1: CDS market-standard quote and upfront**

1. **(a)** MSQ is the conventional quoted spread treated as a zero-upfront par spread for calibrating the flat hazard rate. MSQ is a lesson-local label for the conventional quoted spread. The toy converter treats it as a zero-upfront par spread to infer one flat risk-neutral hazard rate; the contract can have a different fixed running coupon.

2. **(a)** The 100-basis-point standard coupon. The fixed standard coupon determines running premiums. The MSQ calibrates the pricing model, and the upfront amount bridges the resulting protection value and fixed-coupon premium value.

3. 200000 (accepted within $\pm 0.01$). Convert the rates to 0.015 and 0.010. Then U0 = N A (sMSQ - cstd) = 10,000,000 $\times$ 4.0 $\times$ 0.005 = USD 200,000. It is positive, so the protection buyer pays it.

4. $-250000$ (accepted within $\pm 0.01$). U0 = 5,000,000 $\times$ 2.5 $\times$ (0.03 - 0.05) = -USD 250,000. The high fixed coupon makes the running premium leg exceed protection value, so the protection buyer receives the negative signed upfront.

5. 150 (accepted within $\pm 0.001$). With the calibrated annuity held fixed, sMSQ = cstd + U0/(N A) = 0.01 + 160,000/(8,000,000 $\times$ 4.0) = 0.015, or 150 basis points per year. A full upfront-to-quote conversion cannot hold A fixed while the quote-implied hazard rate changes.

6. 300 (accepted within $\pm 0.001$). The root is lambda = 0.05 per year: A = (1 - exp(-0.05))/0.05 = 0.97541151 and A $\times$ (0.60 $\times$ 0.05 - 0.01) = 0.0195082302, matching U0/N. The quote is 0.60 $\times$ 0.05 = 0.03, or 300 basis points per year.

# Chapter 7.3

# Credit curve and market observables

Interactive version: https://danieltuzes.github.io/credit-and-derivatives/cds/credit-curve-and-market-observ

## 7.3.1 What you will be able to do

After this lesson, you should be able to:

- classify each quantity in a standard CDS valuation as market-observed, contractual, conventional, or model-solved;
- explain why the traded upfront alone is enough to determine the pricing hazard rate, and why that is a consequence of risk-neutral pricing rather than an extra assumption;
- read a credit curve as tenor marks together with the survival curve fitted to them, and tell a change of quotation from a curve fit;
- calculate the survival probability implied by fitted hazard segments and check that a tenor reprices to its mark.

## 7.3.2 A traded price is an expectation

The risk-neutral lesson established that, in an arbitrage-free model with the cash account as numeraire, a claim's present-value is the expectation under the risk-neutral-probability-measure $\mathbb{Q}$ of its discounted payoff; see (1.7.2). Shreve states the general discounted-payoff formula [1].

A CDS is such a claim. Let the local cds-buyer-discounted-cash-flows $\Pi^{\text{buyer}}$ be the random sum of the protection buyer's discounted signed cash flows over the life of the contract, before any upfront amount. It is the protection received at the modeled default time minus the scheduled and accrued premium paid. Each cash flow is discounted from its own payment time to valuation-time 0. The present value of the CDS is the risk-neutral expectation of this sum:

$$PV_0 = \mathbb{E}^{\mathbb{Q}}\left[\Pi^{\text{buyer}}\right] \tag{7.3.1}$$

Now add the one fact that the market gives. A new standard contract is traded at its market upfront amount: the buyer pays the cds-upfront-amount $U_0$ at time zero and receives the claim valued by (7.3.1). A trade at the market level has zero present value at inception. So the upfront amount equals the present value:

$$U_0 = \mathbb{E}^{\mathbb{Q}}\left[\Pi^{\text{buyer}}\right] = PV_0^{\text{prot}} - PV_0^{\text{prem}} \tag{7.3.2}$$

The second equality follows from the linearity of expectation: the expectations of the protection cash flows and of the premium cash flows are calculated separately. They are the two positive leg present values of the preceding lessons. Equation (7.3.2) is the same identity as (7.2.2), read in the other direction. In the preceding lesson, the model parameters were inputs and the upfront was the output. Here, the upfront is observed, and the equation constrains the model.

**Why one equality is enough.** The standard contract has no cash flow at inception other than the upfront amount. So the trade gives exactly one number. In the model of this course, discounting and recovery are deterministic, and both legs are linear expectations over the default time. So the only unknown in (7.3.2) is the timing of default under the pricing measure: the survival-probability curve. Setting the expected values of the two legs equal, with the upfront amount on the premium side, is therefore not a modeling choice added to risk-neutral pricing. It is what risk-neutral pricing says about a contract that has just been traded at the market level. Tuckman and Serrat describe the market convention in these steps. First, find the hazard rate that makes the expected discounted fee leg and contingent leg (in this course, the premium leg and the protection leg) equal. Then value the standard coupon at that hazard rate [2].

One number determines one unknown. With a single flat Constant hazard rate $\lambda$ per contract, as in the preceding lesson, one traded price is exactly enough. With a term structure of hazard rates, one traded price per tenor is exactly enough only if the curve has one free parameter per tenor. The piecewise-constant curve below has this property. With fewer prices than parameters, the curve is under-determined, and a convention must fix the remaining parameters. With more prices than parameters, the curve can in general only fit the prices approximately (a best fit).

### 7.3.3 What the market shows, the model assumes, and the model solves

If every quantity is called an input, it is not clear which quantities the market can contradict. Table 7.3.1 classifies the quantities of a standard contract at one tenor.

The risk-neutral interpretation has two consequences.

- **The solved probabilities are pricing weights.** Hull describes implying default probabilities from CDS quotes as the analogue of implying volatilities from option prices. Hull states that CDS valuation needs risk-neutral, not real-world, default probabilities [10]. A survival curve fitted to quotes is a statement about prices, not a forecast of default frequencies. Tuckman and Serrat make the same point for the price-implied hazard rate [11].
- **Recovery must be assumed because one price cannot identify two unknowns.** The protection payoff is proportional to LGD, and the implied hazard rate is approximately proportional to 1/LGD. So a contract valued with the same recovery rate that implied its hazard rate is not very sensitive to the choice of recovery rate. A binary CDS, whose payoff does not depend on recovery, would add the second equation needed to separate the two [12].

### 7.3.4 The credit curve is two objects

A discount curve is a set of dated discount factors in one currency and on one funding basis; it is not one yield (see the rates lesson). A credit curve is the same idea for one reference entity. But a

Table 7.3.1 **Observed, contractual, convention, and solved quantities.** Each quantity of a standard contract at one tenor: whether the market shows it, the contract fixes it, a convention assumes it, or the model solves it, and where it comes from.

| Quantity | Status | Where it comes from |
|---|---|---|
| Signed upfront $U_0$, or the cds-market-standard-quote it stands for | **Observed** | Dealer bid and offer for the standard contract; a mid is taken as the mark. [3] |
| Risk-free discount-factor $D(0,t)$ | **Observed** in another market, then an input | Built from the rates market, not from the CDS; the converter takes it as a discount input. [4] |
| cds-standard-coupon $c_{\text{std}}$, cds-notional $N$, maturity, credit events, reference obligations | **Contractual** | Written into the standard contract; the coupon is 100 or 500 basis points and maturities fall on standard dates. [5] [6] |
| recovery-rate $R$ and hence loss-given-default LGD | **Convention** | Assumed by the converter, commonly 40% but 20% or 25% for some contract types. It is not observed in the quote. [7] |
| Shape of the hazard rate between and beyond tenors, accrual treatment | **Convention** | The converter's single flat hazard rate, or this lesson's piecewise-constant curve. [8] |
| Constant hazard rate $\lambda$, the survival curve, the cds-par-spread $s^\star$, the other quotation | **Solved** | Root of (7.3.2) given everything above. [9] |

credit curve has two layers, and this course keeps them separate.

**The marked curve.** Use the local cds-tenor-index $k$ across the cds-tenor-count $K$ quoted tenors with maturities $T_1 < \cdots < T_K$. At each tenor, the market shows the price of the standard contract. A mark is the recorded price at one tenor. If the trading desk records the price as a conventional spread, the mark is the local cds-tenor-quote $s_k$. If the desk records the traded price itself, the mark is the local cds-tenor-upfront-fraction $u_k$. This course calls a curve whose marks are market-standard quotes an **MSQ-marked curve**, and one whose marks are upfront fractions an **upfront-marked curve**. Both labels are local to this course (`NEEDS_SOURCE`: the desk vocabulary for marking conventions is not documented in the registered sources). Tuckman and Serrat tabulate five-year spreads for several reference entities. They also plot the term structure of spreads across tenors: its slope is normally upward, but it is inverted in periods of stress [13]. Hull lists the tenors that commonly trade; the five-year tenor is the most liquid [14].

**The fitted curve.** The legs lesson allows a deterministic time-varying hazard rate. It shows that the survival probability is the exponential of minus the integral of the hazard rate. The fitted credit curve is the special case in which the hazard rate is constant on each segment between consecutive tenors: the local cds-segment-hazard-rate $\lambda_k$ on $(T_{k-1}, T_k]$, with $T_0 = 0$. The survival probability is then a product with one factor per segment:

$$S^{\mathbb{Q}}(0,T_k) = S^{\mathbb{Q}}(0,T_{k-1}) \exp\Big(-\lambda_k \left(T_k - T_{k-1}\right)\Big), \qquad S^{\mathbb{Q}}(0,0) = 1 \tag{7.3.3}$$

Inside a segment, the survival probability decreases at the constant hazard rate of that segment, as in the constant-hazard lesson. Hull notes that when spreads for different maturities are available, the hazard rate is calculated as a step function [15].

The marked curve is data. The fitted curve is a model object, chosen so that the model reprices the data. The next distinction depends on keeping the two layers separate.

### 7.3.5 Transforming marks is not fitting the curve

**Curve transformation** changes the quotation of each mark and does not build a term structure. For each tenor separately, the standard converter takes $s_k$ and solves for a flat hazard rate for that tenor alone. This course writes this rate as the local cds-flat-tenor-hazard-rate $\lambda_k^{\text{flat}}$. The converter then values the fixed coupon at that rate and returns $u_k$. The converter can also do the same steps in the reverse direction. This is the conversion of the preceding lesson, applied once per tenor [16]. The flat hazard rates of different tenors are solutions of separate equations. They are not one survival curve, and the official converter itself distinguishes its single flat hazard rate from a term structure [17]. The same tenor-by-tenor conversion also gives a third quotation: the par spread of a hypothetical zero-upfront contract. The name **curve transformation** belongs to this course (`NEEDS_SOURCE` for the phrase itself).

**Fitting the curve** finds one term structure that reprices every mark at once. With the marks written as upfront fractions, the fit condition at tenor $k$ is the buyer-value identity (7.3.2), evaluated with the segment hazard rates up to $k$:

$$PV_0^{\text{prot}}(T_k; \lambda_1, \dots, \lambda_k) - c_{\text{std}} N A_0^{\text{prem}}(T_k; \lambda_1, \dots, \lambda_k) = u_k N \tag{7.3.4}$$

Both legs are the exact default-time integrals of the legs lesson, with the piecewise-constant hazard rate substituted. Each premium period lies inside one segment. So each period is a one-period flat-hazard model that starts from the survival probability and the discount factor at the start of the period. Solve (7.3.4) in maturity order. The first tenor determines $\lambda_1$. Then $\lambda_1$ is held fixed, and the second tenor determines $\lambda_2$, and so on. Each step is a one-dimensional root solve for $\lambda_k$. The buyer value of a contract increases with the hazard rate of any segment inside the life of the contract, so each step has at most one root. A zero-upfront par mark is the special case $u_k = 0$, with the coupon set to the quoted par spread. The result is a curve under which every quoted tenor reprices exactly. The survival probabilities of this curve at all tenors are consistent with each other.

The same survival curve then prices every other contract on the same reference entity: a maturity between the quoted tenors, an existing contract with a different coupon, or a hypothetical zero-upfront contract. This is the purpose of a fitted curve. The fit has two limits. First, the fitted par spread at a tenor is in general not equal to the MSQ mark of that tenor. The reason is that the fitted premium annuity differs from the flat annuity of the converter, although both reprice the same traded upfront amount. This is the "par spread from another curve" point of the preceding lesson. Second, a solution need not exist. A later mark that is cheaper than the zero-hazard value implied by the earlier segments would need a negative $\lambda_k$. The model rejects a negative hazard rate, because a survival probability cannot increase with time.

**Example 7.3.1: A credit curve from market marks**

**Transform the marks**

Take an MSQ-marked curve for one reference entity: standard contracts with quarterly premiums and a fixed coupon $c_{\text{std}} = 0.010$ per year (100 basis points per year), continuously compounded risk-free rate 2% per year, recovery $R = 0.40$, and marks $s_1 = 0.008$, $s_2 = 0.012$, $s_3 = 0.016$ at maturities $T_1 = 1$, $T_2 = 3$, $T_3 = 5$ model-years. Notional is USD 10,000,000.
For each tenor separately, the flat converter of the preceding lesson gives:

| Tenor $k$ | $s_k$ (bp/yr) | $\lambda_k^{\text{flat}}$, per yr | Flat $A_0^{\text{prem}}$, model-yrs | $u_k$ | $U_0$, USD | Flat $S(0,T_k)$ |
|---|---|---|---|---|---|---|
| 1 | 80 | 0.013300046 | 0.981077842 | −0.001962156 | −19,621.56 | 0.986788009 |
| 2 | 120 | 0.019950083 | 2.820140305 | 0.005640281 | 56,402.81 | 0.941905574 |
| 3 | 160 | 0.026600129 | 4.449093452 | 0.026694561 | 266,945.61 | 0.875464525 |

Each row is the conversion of the preceding lesson. For example, at five years, $u_3 = 4.449093452 \times (0.016 - 0.010) = 0.026694561$. The one-year mark is below the coupon, so the buyer receives the upfront amount. The three flat hazard rates are solutions of three separate equations. The last column is not one survival curve: for example, it does not define the survival probability at two years.

**Fit the curve**

Now fit a piecewise-constant hazard curve to the three upfront fractions in maturity order.

| Tenor $k$ | Segment, model-yrs | $\lambda_k$, per yr | Fitted $S(0,T_k)$ | Fitted $A_0^{\text{prem}}$, model-yrs | Fitted $PV_0^{\text{prot}}/N$ | Fitted $s^\star$ (bp/yr) |
|---|---|---|---|---|---|---|
| 1 | (0, 1] | 0.013300046 | 0.986788009 | 0.981077842 | 0.007848623 | 80.0000 |
| 2 | (1, 3] | 0.023463765 | 0.941550234 | 2.829254583 | 0.033932826 | 119.9356 |
| 3 | (3, 5] | 0.037572831 | 0.873389867 | 4.500220799 | 0.071696769 | 159.3183 |

Every tenor reprices. For example, at five years, $0.071696769 - 0.010 \times 4.500220799 = 0.026694561 = u_3$. In the tested implementation, the residuals at all three tenors are below $10^{-12}$.

Compare this table with the transformation. The hazard rate of the first segment equals the one-year flat rate, because a one-year contract depends only on the first segment. The hazard rates of the later segments are higher than the flat rates. The five-year flat rate, 0.026600, is an average over five years. The fitted hazard rate on the last two years is 0.037573, because the shorter tenors, with lower hazard rates, already price the earlier years. So the fitted five-year survival probability, 0.873390, is below the flat survival probability, 0.875465. The fitted par spreads at three and five years, 119.94 and 159.32 basis points, are not equal to the marks of 120 and 160 basis points. The reason is that the fitted curve reproduces the same traded upfront amounts with different premium annuities.

The fitted curve also defines the survival probability between tenors. By (7.3.3), the survival probability at two years is $0.986788009 \exp(-0.023463765 \times 1) = 0.963903771$, and at four years it is $0.941550234 \exp(-0.037572831 \times 1) = 0.906829881$.

**The marks are prices of particular contracts**

Now read the same three numbers, 80, 120, and 160 basis points, as zero-upfront par spreads, not as MSQ marks with a coupon of 100 basis points. So $u_k = 0$, and the coupon of each tenor equals its quote. The fit then gives segment hazard rates of 0.013300046, 0.023480220, and 0.037863368 per year, and a five-year survival probability of 0.872853786 instead of 0.873389867. The first segment is unchanged, because the converter also treats the one-year MSQ of 80 basis points as the par spread of a zero-upfront one-year contract. The later segments change. A standard contract with a coupon of 100 basis points and an upfront amount is a different stream of cash flows from a zero-upfront contract with a coupon of 120 basis points. The two prices are written with the same number, but they are prices of different contracts. A mark is the price of a particular contract, and the fit must know which contract.

**When no curve fits**

Suppose the one-year zero-upfront par mark is 300 basis points and the three-year mark is 100 basis points. The first segment prices the one-year contract. Then the three-year contract at

100 basis points is worth more to the seller than the first year alone implies, even with zero default risk after year one. So its mark is below the zero-hazard bound, and the fit reports that the mark would need a negative segment hazard rate. In the tested implementation, a three-year mark of 110 basis points has a solution, with a hazard rate of 0.001573 per year in the second segment.
Steeply inverted marks are not necessarily an arbitrage, but they indicate a problem. Either the marks are out of date or not at market levels, or the piecewise-constant shape and the deterministic recovery of the model cannot describe them.

### 7.3.6 A note on jargon

The vocabulary of CDS trading desks mixes contract terms, quotation conventions, and risk measures. Table 7.3.2 gives this course's reading of the most common phrases, with the registered source where one supports it. Entries marked `NEEDS_SOURCE` are recorded as usage this course adopts locally; a human should confirm or replace them before review.

### 7.3.7 Check your understanding

The assessment separates the observed-versus-solved classification, the transformation-versus-fit distinction, and the survival and repricing arithmetic of a fitted curve. Each competency has direct and transfer evidence.

**Knowledge check 7.3.1: Credit curve and market observables**

1. In the standard conversion of a five-year contract's quote into an upfront, which quantity is solved from the traded price rather than observed in the market or assumed by convention?

   (a) The flat risk-neutral hazard rate

   (b) The fixed running coupon

   (c) The recovery rate

2. A dealer shows five-year protection at 160 basis points bid and 165 basis points offer, and the converter uses a 40% recovery rate. Which statement is correct?

   (a) The 40% recovery is a convention assumed by the converter; the quote does not observe it

   (b) The quote reveals that the market expects 40% recovery

   (c) The dealer is quoting the hazard rate directly

3. An MSQ-marked curve lists 80, 120, and 160 basis points at one, three, and five years. Converting each mark to an upfront with the standard converter, one tenor at a time, is best described as which of the following?

   (a) A change of quotation using a separate flat hazard rate per tenor; it does not build one term structure

   (b) A fit of one piecewise-constant hazard curve that reprices all three tenors together

   (c) A direct observation of the survival probabilities at one, three, and five years

Table 7.3.2 **CDS desk jargon.** Common phrases of CDS trading desks with this course's reading of each and the registered source that supports it. Entries marked NEEDS_SOURCE are local usage for a human to confirm.

| Phrase | Meaning in this course |
|---|---|
| Reference entity, credit event, notional | The named borrower, the contractually defined trigger, and the amount protected. [18] [19] |
| Buying protection, selling protection; short credit, long credit | A protection buyer pays premium and gains on default, like a short seller of the bond; a protection seller resembles a bond holder. [20] |
| Index buyer and seller | For index CDS the words are reversed: the index buyer receives premium and pays on default, like a portfolio holder. [21] |
| Spread, quoted spread, conventional spread, MSQ | The annualized premium of a hypothetical zero-upfront contract, used as the conventional quote; this course's local label is market-standard quote. [22] [14] |
| Par spread | The running spread that makes the two legs equal with zero upfront under a stated curve; see the cds-par-spread card. |
| Standard coupon, points upfront, price | The fixed 100 or 500 basis point running coupon and the upfront that balances it; the upfront per 100 notional is quoted as points, or as a price of 100 minus points. [23] [24] |
| Risky annuity, risky duration, RPV01 | The premium annuity: the present value of one unit of running spread per unit notional, survival-weighted; Hull calls it the duration of the CDS payments. The label RPV01 is `NEEDS_SOURCE`. [24] |
| Risky DV01, spread DV01, CS01 | The change in a CDS value for a one basis point change in the CDS spread, found by re-solving the hazard rate and revaluing; the next lesson uses it. The label CS01 is `NEEDS_SOURCE`. [25] |
| Duration times spread, DTS | A spread-risk measure for bonds that scales spread duration by the spread level, motivated by spreads moving proportionally. [25] |
| On-the-run, off-the-run | The most liquid current standard contract of a tenor versus earlier contracts with older maturity dates. [26] |
| IMM dates, roll | Standard maturity dates on the twentieth of March, June, September, and December; contracts and index series move to new dates on a schedule. [23] [27] |
| Recovery convention | The assumed recovery in the converter: commonly 40%, with 20% or 25% for some contract types. [7] |
| Credit curve, spread curve, term structure of spreads | Marks across tenors and the survival curve fitted to them; upward-sloping in calm markets and inverted in stress. [21] |
| MSQ-marked curve, upfront-marked curve, curve transformation, curve fitting | Local labels defined in this lesson (`NEEDS_SOURCE` for desk usage). |
| Equivalent notional, equivalent ratio | The liquid-tenor notional whose quote sensitivity offsets a position, and its ratio to the position notional; defined in the next lesson (`NEEDS_SOURCE` for desk usage). |
| CDS-bond basis, negative basis trade | The CDS spread minus a bond spread measure; a negative basis trade buys the bond and buys protection. [28] [29] |
| CDS-equivalent bond spread, hazard-adjusted duration | The CDS spread corresponding to the hazard rate that reprices a bond, and a bond duration computed under the hazard model. [30] |
| Cheapest-to-deliver, auction, final price | The delivery option among reference obligations and |

4. Under a fitted piecewise-constant curve, the three-year contract's par spread is 119.94 basis points while its MSQ mark is 120 basis points, and the fit reprices the three-year upfront exactly. Why can both numbers be right?

   (a) They are par spreads under two different survival curves that reprice the same traded upfront with different premium annuities

   (b) The fit failed to reprice the three-year tenor

   (c) The difference is an arbitrage between the two quotations

5. Under the fitted curve, the five-year tenor has protection factor 0.071696769 per unit notional and premium annuity 4.500220799 model-years, and its standard coupon is 100 basis points per year. Calculate the upfront fraction of notional, paid by the protection buyer, that the fit reproduces.

6. A fitted curve has a segment hazard rate of 0.02 per year on (0, 2] and 0.05 per year on (2, 5] model-years. Calculate the survival probability at four years.

*Answers: end of the chapter.*

### 7.3.8 Model boundary and review note

This AI-assisted lesson and all cited locators remain `draft` pending human editorial and quantitative review. All input values are invented for teaching. The fitted curve is piecewise constant with knots at quoted tenors, extended flat, under one deterministic discount rate, one deterministic recovery rate, and exact default-time accrual on a regular model-year schedule. A production curve also handles calendar schedules, day counts, business-day rules, settlement timing, accrued premium at inception, an interpolation and extrapolation policy chosen by the trading desk, and a discount curve built from the rates market. The lesson does not model bid-offer, liquidity, dealer inventory, stochastic rates or intensity, recovery risk, counterparty risk, collateral, funding, or transaction costs, and it makes no claim that pricing-model survival probabilities forecast realized defaults.

## References

[1] Shreve, Stochastic Calculus for Finance II: Continuous-Time Models (2004). Ch. 5 §5.2.4, printed pp. 218-219, eqs. 5.2.29-5.2.31.

[2] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.6, printed pp. 368-370.

[3] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.1, printed p. 549 (market-maker bid and offer in basis points per year).

[4] International Swaps and Derivatives Association, ISDA CDS Standard Model and Standard CDS Contract Converter Specification. Standard CDS Contract Converter Specification, version May 5, 2009, Functionality and Specification, printed p. 1. https://www.cdsmodel.com/documentation.html

[5] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.6, printed pp. 366-367 (standard maturity dates on the twentieth of March, June, September, and December; 100 or 500 basis point coupons with a market-determined upfront).

[6] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.4, printed p. 556 (a coupon and a recovery rate are specified for each underlying and maturity).

[7] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.6, printed p. 368, n. 15.

[8] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.6, printed p. 370 (the quote-to-upfront translation is a convention like price and yield; participants need not believe in constant hazard or a particular recovery, but all use the convention).

[9] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.4, printed pp. 556-557 (a hazard rate is implied from the quoted spread by iterative search).

[10] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.2, printed p. 554.

[11] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.7, printed p. 371.

[12] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.2, printed pp. 554-555.

[13] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.5, printed pp. 362-363, Table 14.9 and Figure 14.5.

[14] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.1, printed p. 549.

[15] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.2, printed p. 554, n. 6.

[16] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.4, printed pp. 556-557.

[17] International Swaps and Derivatives Association, ISDA CDS Standard Model and Standard CDS Contract Converter Specification. Standard CDS Examples, version October 26, 2012, printed p. 2 (the converter uses a single flat hazard rate rather than a term structure of flat spreads). https://www.cdsmodel.com/documentation.html

[18] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.1, printed pp. 548-549.

[19] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.5, printed p. 361.

[20] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.5, printed pp. 364-365.

[21] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.5, printed p. 363.

[22] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.5, printed p. 362.

[23] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.6, printed p. 366.

[24] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.4, printed p. 557.

[25] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.10, printed p. 379.

[26] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.6, printed pp. 366-367.

[27] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.3, printed pp. 555-556.

[28] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.8, printed pp. 374-375.

[29] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.1, printed pp. 550-551.

[30] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.7, printed p. 371; §14.9, printed p. 377.

[31] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.1, printed pp. 549 and 551.

[32] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.5, printed p. 363; §14.13, printed p. 387.

[33] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.3, printed p. 555.

[34] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.6, printed p. 367, n. 14.

[35] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.1, printed p. 550.

## Answers to the knowledge checks

**Knowledge check 7.3.1: Credit curve and market observables**

1. **(a)** The flat risk-neutral hazard rate. The coupon is contractual and the recovery rate is a convention assumed by the converter. The hazard rate is the unknown that the one traded price identifies through the equality of expected discounted leg values.

2. **(a)** The 40% recovery is a convention assumed by the converter; the quote does not observe it. One traded price cannot identify both recovery and default timing, so the converter assumes recovery and solves the hazard rate. The bid and offer are the observed market levels; the recovery is not among them.

3. **(a)** A change of quotation using a separate flat hazard rate per tenor; it does not build one term structure. The tenor-by-tenor conversion is a curve transformation. Each tenor's flat hazard rate answers a separate question, so the three flat rates are not one survival curve; fitting the curve is a separate step that solves segment hazards in maturity order.

4. **(a)** They are par spreads under two different survival curves that reprice the same traded upfront with different premium annuities. The MSQ mark is the par spread under the converter's single flat hazard for that tenor; the fitted par spread uses the term structure's premium annuity. Both reproduce the same traded upfront, and the hypothetical zero-upfront contract is not itself traded, so there is no arbitrage.

5. 0.026694561 (accepted within $\pm 0.000001$). The fit condition per unit notional is protection factor minus coupon times premium annuity: $0.071696769 - 0.010 \times 4.500220799 = 0.026694561$, which equals the five-year mark.

6. 0.869358235 (accepted within $\pm 0.000001$). Survival compounds segment by segment: $\exp(-0.02 \times 2) \times \exp(-0.05 \times 2) = \exp(-0.14) = 0.869358235$. Using only the second segment's rate over four years would be wrong.

# Chapter 7.4

# Equivalent notional and quote risk

Interactive version: https://danieltuzes.github.io/credit-and-derivatives/cds/equivalent-notional-and-quote-

## 7.4.1 What you will be able to do

After this lesson, you should be able to:

- calculate a CDS position's signed value change for a one basis point increase in the liquid tenor's quote, re-solving the hazard rate rather than holding the premium annuity fixed;
- explain why, in a one-factor model, risk measured against the market-standard quote, the upfront, or the par spread is the same risk;
- calculate the equivalent notional and side of the most liquid contract that offsets a position, and its equivalent ratio;
- state what an equivalent-ratio hedge does not offset.

## 7.4.2 One market level moves everything

The preceding lessons value every contract on a reference entity with one flat Constant hazard rate $\lambda$. The quote lesson showed how the market level determines $\lambda$: the cds-market-standard-quote $s_{\mathrm{MSQ}}$ of the standard contract is treated as a par spread, and $\lambda$ is the root of (7.2.4). In this lesson, that contract is the most liquid one, with the local cds-liquid-tenor-maturity $T_L$. In practice, the most liquid tenor is usually five years [1] [2].

A position is one contract on the same reference entity with its own maturity, fixed coupon, and positive local cds-position-notional $N_{\mathrm{pos}}$. Its legs are the exact default-time expectations of the legs lesson, evaluated at the same $\lambda$. Let the local cds-position-side $\sigma$ be $+1$ for bought protection and $-1$ for sold protection. The local cds-position-value is the protection-buyer value of the legs lesson, multiplied by the side:

$$V^{\mathrm{pos}} = \sigma \left( PV_0^{\mathrm{prot}} - PV_0^{\mathrm{prem}} \right) \tag{7.4.1}$$

Hull marks a CDS to market in the same way, with opposite signs for buyer and seller [3]. Any upfront exchanged at inception is a sunk cash flow and is left out of $V^{\mathrm{pos}}$; it does not change any sensitivity.

For the liquid contract, the local cds-liquid-unit-value is the buyer value per unit of notional:

$$v_L = \frac{PV_0^{\text{prot}}(T_L)}{N} - c_{\text{std}} A_0^{\text{prem}}(T_L) \tag{7.4.2}$$

This value is the fair upfront fraction of the quote lesson. So at the current market level, it equals the upfront amount of the liquid contract per unit of notional [4].

Because $\lambda$ is the only state variable, every quotation of the liquid contract is a function of $\lambda$. Write the local cds-liquid-quotation $q$ for any one of them: the market-standard quote, the upfront fraction, or the par spread. In this model, each quotation is strictly increasing in $\lambda$, so each quotation is invertible. So a change of the quote is a change of the hazard rate, whichever quotation the trading desk uses for its marks. The model has one risk factor, expressed in three quotations. In this model, with a flat hazard rate and a flat interest rate, the par spread is also the same at every maturity. So the par spread of a position and the liquid quote are the same number.

### 7.4.3 Risky DV01 by bump and reprice

Tuckman and Serrat define the risky DV01 of a CDS as the change in its value for a one basis point change in the CDS spread, computed by finding the hazard rate that moves the spread by one basis point and revaluing the contract [5]. This lesson uses that bump-and-reprice definition with the local cds-quote-bump $h = 0.0001$ per year, one basis-point of market-standard quote.

Write $\lambda(s_{\text{MSQ}})$ for the hazard rate re-solved from a quote. The local cds-position-quote-sensitivity is the change in the position value when the quote increases by $h$:

$$\text{DV01}^{\text{pos}} = V^{\text{pos}}\Big(\lambda\left(s_{\text{MSQ}} + h\right)\Big) - V^{\text{pos}}\Big(\lambda\left(s_{\text{MSQ}}\right)\Big) \tag{7.4.3}$$

The local cds-liquid-unit-quote-sensitivity is the same difference, for one unit of bought liquid protection:

$$\text{DV01}^{L} = v_L\Big(\lambda\left(s_{\text{MSQ}} + h\right)\Big) - v_L\Big(\lambda\left(s_{\text{MSQ}}\right)\Big) \tag{7.4.4}$$

Three properties of the definition are important in practice.

- **Both legs change.** A higher hazard rate increases the protection leg and decreases the premium annuity. So the sensitivity is not the annuity multiplied by one basis point. Holding the annuity fixed is the shortcut that the quote lesson warned against.
- **The sign depends on the side.** Bought protection gains value when the quote increases, so its sensitivity is positive. Sold protection has a negative sensitivity. $\text{DV01}^{L}$ is defined for bought protection and is positive.
- **The bump is finite.** The value is a nonlinear function of the quote. So the result of a one basis point bump differs slightly from the derivative. The section “What an equivalent-ratio hedge does not do” discusses this difference.

### 7.4.4 The liquid-tenor equivalent

A hedge that removes the quote risk is a position in the liquid contract whose notional makes the sensitivity of the hedged position zero. Let the local cds-equivalent-notional $N_{\text{eq}}$ be the notional of bought liquid protection. The hedge condition and its solution are:

$$\mathrm{DV01}^{\mathrm{pos}} + N_{\mathrm{eq}}\,\mathrm{DV01}^{L} = 0 \qquad \Longleftrightarrow \qquad N_{\mathrm{eq}} = -\frac{\mathrm{DV01}^{\mathrm{pos}}}{\mathrm{DV01}^{L}} \tag{7.4.5}$$

A positive $N_{\mathrm{eq}}$ means: buy protection in the liquid tenor. A negative $N_{\mathrm{eq}}$ means: sell protection in the liquid tenor. Because $\mathrm{DV01}^{L}$ is positive, the sign of $N_{\mathrm{eq}}$ is always opposite to the sign of the position's sensitivity. The local cds-equivalent-ratio is the size of the hedge relative to the position:

$$\rho_{\mathrm{eq}} = \frac{|N_{\mathrm{eq}}|}{N_{\mathrm{pos}}} \tag{7.4.6}$$

This hedge is the DV01-neutral hedge that Tuckman and Serrat describe for bonds. There, the face amount of a liquid hedge is minus the ratio of the two DV01s, multiplied by the position amount. The instrument with the larger sensitivity is traded in the smaller quantity [6]. The names **equivalent notional** and **equivalent ratio**, for the hedge amount and its ratio, belong to this course (`NEEDS_SOURCE` for desk usage; the registered sources define the DV01 hedge but not these names).

**The choice of quotation does not change the hedge.** A trading desk can measure the position's risk against the market-standard quote, the upfront, or a par spread. In this model, the choice does not change the equivalent notional. In the limit of a small bump, both sensitivities are derivatives with respect to the same quotation $q$. The chain rule through the one state variable $\lambda$ gives:

$$N_{\mathrm{eq}} = -\frac{dV^{\mathrm{pos}}/dq}{dv_L/dq} = -\frac{(dV^{\mathrm{pos}}/d\lambda)\,(d\lambda/dq)}{(dv_L/d\lambda)\,(d\lambda/dq)} = -\frac{dV^{\mathrm{pos}}/d\lambda}{dv_L/d\lambda} \tag{7.4.7}$$

The factor $d\lambda/dq$, which converts a change of the hazard rate into a change of the quotation, cancels. So MSQ risk, upfront risk, and par spread risk are one risk, and the same equivalent notional offsets all three. The sensitivities themselves differ across quotations, because each sensitivity is measured per unit of a different quotation. The hedge amount is the same. With a finite bump, the three equivalent ratios differ only because of the curvature, as the worked example shows.

**Example 7.4.1: Equivalent-notional hedges**

**A seven-year position hedged with five-year**

The liquid contract is a five-year standard contract with quarterly premiums, $c_{\mathrm{std}} = 0.010$ per year (100 basis points per year) and market-standard quote $s_{\mathrm{MSQ}} = 0.016$ per year (160 basis points per year). The risk-free rate is 2% per year, continuously compounded, and the recovery rate is 40%. Calibration to the quote gives $\lambda = 0.026600129$ per year. By (7.4.2), one unit of bought five-year protection is worth $v_L = 0.026694561$ before the upfront amount.

The position is bought protection on a seven-year contract with the same coupon of 100 basis points and $N_{\mathrm{pos}} =$ USD 10,000,000. At the calibrated hazard rate, the protection-leg present value of the position is USD 953,285.99, and its premium-leg present value is USD 595,803.74. So by (7.4.1), $V^{\mathrm{pos}} =$ USD 357,482.24.

Bump the liquid quote by $h = 0.0001$, to 161 basis points. The re-solved hazard rate is 0.026766381 per year. Revaluing with (7.4.3) and (7.4.4) gives the two sensitivities:

$$\mathrm{DV01}^{\mathrm{pos}} = 363{,}240.36 - 357{,}482.24 = \text{USD } 5{,}758.12 \text{ per basis point}$$

$$\mathrm{DV01}^{L} = 0.000434070474 \text{ per unit notional per basis point}$$

By (7.4.5), the equivalent notional is:

$$N_{\text{eq}} = -\frac{5{,}758.117764}{0.000434070474} = -\text{USD } 13{,}265{,}398.37$$

So the hedge sells protection on a notional of USD 13,265,398 of the five-year contract. By (7.4.6), $\rho_{\text{eq}} = 1.3265$. The seven-year contract has the larger sensitivity per unit of notional. So the hedge notional in the five-year contract is larger than the notional of the position. The tested implementation reports a hedged sensitivity of zero to arithmetic precision.

**A sold three-year position at a 500 basis point coupon**

Keep the same liquid contract and market level. The position is sold protection on a three-year contract with coupon 500 basis points per year and $N_{\text{pos}} = \text{USD } 20{,}000{,}000$, so $\sigma = -1$. Because the coupon of 500 basis points is higher than the market level of 160 basis points, the seller's value is positive: $V^{\text{pos}} = \text{USD } 1{,}899{,}072.40$.
After the same bump of one basis point, the seller's value decreases to USD 1,893,025.81. So the sensitivity is:

$$\text{DV01}^{\text{pos}} = -\text{USD } 6{,}046.59 \text{ per basis point}$$

The seller loses value when the quote increases. With the same $\text{DV01}^{L} = 0.000434070474$, the equivalent notional is:

$$N_{\text{eq}} = -\frac{-6{,}046.591823}{0.000434070474} = +\text{USD } 13{,}929{,}977.23$$

So the hedge buys protection on a notional of USD 13,929,977 of the five-year contract, and $\rho_{\text{eq}} = 0.6965$. The three-year contract has a smaller sensitivity per unit of notional than the five-year contract. So a five-year notional smaller than the position notional offsets the position. This holds although the high coupon gives the position a large value.

**Three quotations, one hedge**

Repeat the seven-year hedge, but bump the upfront fraction of the liquid contract by 0.0001 (one basis point of notional) instead of its quote. Re-solve the hazard rate from the bumped upfront fraction. The equivalent ratio is 1.32670, compared with 1.32654 from the quote bump. With both bumps one thousand times smaller, both ratios are 1.32674, as (7.4.7) predicts. So the difference at one basis point is a result of the curvature of a finite bump, not of a different risk.
As a check, apply the construction to the liquid contract itself, with bought protection on USD 10,000,000. The equivalent ratio is exactly one, on the opposite side: the hedge is the offsetting trade.

### 7.4.5 What an equivalent-ratio hedge does not do

The hedge sets one sensitivity to zero, at one market level, in one model. It leaves at least the following exposures, and a trading desk manages each of them separately.

- **Default.** The seven-year hedge above sells protection on USD 13,265,398 against bought protection on USD 10,000,000. If default occurs immediately, with 40% recovery, the position receives $\text{LGD} \times 10{,}000{,}000 = \text{USD } 6{,}000{,}000$ and the hedge pays $0.6 \times 13{,}265{,}398.37 = \text{USD } 7{,}959{,}239.02$. The net loss is about USD 1,959,239, before accrued premium. Equal

quote sensitivities do not imply equal default payoffs. So the jump-to-default exposure of a hedged portfolio must be measured separately.

- **Curve shape.** The model has one hazard rate. Under the fitted curve of the credit-curve lesson, a seven-year position also depends on the segments beyond five years, on which the five-year contract does not depend. A change of the five-year quote alone would change the value of only part of the position. Measuring the sensitivity separately for each tenor is the standard extension (`NEEDS_SOURCE`: the registered sources describe single-factor DV01 hedging only).
- **Curvature.** Both sensitivities change when the quote changes. So the hedge stops being exact and must be rebalanced. Tuckman and Serrat make the same point for bond DV01 hedges [7].
- **Inputs held fixed.** The recovery rate, the discount curve, and the accrual and settlement conventions are constant in the model. A change in any of them changes the values of the two contracts by different amounts.
- **Costs.** The hedge pays the bid-offer spread of the liquid contract. The lower liquidity of the position's own tenor is the reason why the hedge is not the offsetting trade in that tenor [8].

### 7.4.6 Check your understanding

The assessment separates the bump-and-reprice sensitivity, including its sign for a seller, from sizing the equivalent notional and ratio. Each competency has direct and transfer evidence.

**Knowledge check 7.4.1: Equivalent notional and quote risk**

1. A bought-protection position is worth USD 357,482.24 at the liquid five-year quote of 160 basis points and USD 363,240.36 after the quote is bumped to 161 basis points with the flat hazard rate re-solved. Calculate the position's risky DV01 in USD per basis point.
2. A trader has sold protection on a three-year contract. Valued for a protection buyer, the contract is worth USD -1,899,072.40 at the liquid quote and USD -1,893,025.81 after a one basis point bump with the hazard rate re-solved. Calculate the seller's risky DV01 in USD per basis point, using the sign convention that a positive number is a gain for the seller.
3. A bought-protection position has risky DV01 of USD 5,758.117764 per basis point. One unit of notional of bought protection in the liquid five-year contract has risky DV01 of 0.000434070474 per basis point. Calculate the magnitude in USD of the liquid-tenor equivalent notional that offsets the position.
4. A sold-protection position with notional USD 20,000,000 has risky DV01 of USD -6,046.591823 per basis point for the seller. One unit of notional of bought protection in the liquid five-year contract has risky DV01 of 0.000434070474 per basis point. Calculate the equivalent ratio of the liquid-tenor hedge to the position notional.

*Answers: end of the chapter.*

### 7.4.7 Model boundary and review note

This AI-assisted lesson and all cited locators remain `draft` pending human editorial and quantitative review. All input values are invented for teaching. The sensitivity is a one basis point bump of the liquid tenor's market-standard quote, under one flat hazard rate, one deterministic discount rate, one deterministic recovery rate, and a regular quarterly model-year schedule. The hedge is sized at one market level and is not rebalanced. The lesson omits curve shape risk, default and recovery risk beyond the illustration above, bid-offer, transaction costs, calendar and settlement conventions, counterparty risk, collateral, funding, and any stochastic model of rates or intensity.

## References

[1] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.1, printed p. 549.

[2] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.5, printed p. 365.

[3] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.2, printed pp. 553-554.

[4] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.4, printed p. 557.

[5] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.10, printed p. 379.

[6] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 4 §4.3, printed pp. 110-111, eqs. 4.6-4.9.

[7] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 4 §4.2, printed p. 109 (DV01 as a local measure).

[8] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.6, printed pp. 366-367 (unwinding through the on-the-run contract and managing the maturity and spread mismatch).

## Answers to the knowledge checks

**Knowledge check 7.4.1: Equivalent notional and quote risk**

1. 5758.12 (accepted within $\pm 0.01$). The bump-and-reprice sensitivity is the value after the bump minus the value before it: 363,240.36 - 357,482.24 = USD 5,758.12 per basis point. It is positive because bought protection gains when the quote rises.

2. $-6046.59$ (accepted within $\pm 0.01$). The seller's value is minus the buyer's value: USD 1,899,072.40 before and USD 1,893,025.81 after the bump. The seller's sensitivity is 1,893,025.81 - 1,899,072.40 = USD -6,046.59 per basis point; a seller loses when the quote rises.

3. 13265398.4 (accepted within $\pm 1$). $N_{\text{eq}} = \frac{-5{,}758.117764}{0.000434070474} = -$USD 13,265,398.4. The negative sign means the hedge sells protection in the five-year contract; its magnitude is USD 13,265,398.4.

4. 0.6965 (accepted within $\pm 0.0001$). $N_{\text{eq}} = \frac{-(-6{,}046.591823)}{0.000434070474} = +\text{USD } 13{,}929{,}977.2$, so the hedge buys protection in the five-year contract. The equivalent ratio is 13,929,977.2 / 20,000,000 = 0.6965.

# Appendix A

# Notation

The symbols of this book, in the order they first appear.

$A_0^{\text{accr}}$ **CDS accrued premium annuity**. Present-value factor for exact premium accrual paid at modeled default. It integrates accrued year fraction, discounting, and risk-neutral default density over every possible default time. *Units:* model-years. *In Chapter 7.1 only.*

$a_i(u)$ **CDS accrued year fraction at default**. Year fraction accrued from the preceding premium date to a possible default time in period i. In the regular model-year schedule it equals elapsed model time since the period start. *Units:* model-years of premium accrual. *In Chapter 7.1 only.*

$$a_i(u) = u - t_{i-1}$$

$\kappa$ **CDS combined decay rate**. Abbreviation for the sum of the deterministic risk-free rate and constant hazard rate. It shortens the closed forms and has no separate market interpretation. *Units:* decimal rate per model-year. *In Chapter 7.1 only.*

$$\kappa = r + \lambda$$

$G_i$ **CDS default event**. Event that modeled default occurs after one premium date and no later than the next. Together with survival through maturity, these events are disjoint and exhaustive in the one-default model. *In Chapter 7.1 only.*

$$G_i = \{t_{i-1} < \tau \leq t_i\}$$

$u$ **CDS default integration time**. Integration variable ranging over every possible default time inside one CDS period, or over the interval used to cumulate a time-varying hazard rate into a survival probability. It is a deterministic dummy variable; the random default time is a different symbol. *Units:* model-years after valuation time. *In Chapter 7.1 only.*

$Y$ **CDS event payoff**. Random discounted payoff used to connect event partitions to a CDS-leg expectation. Its currency sign and perspective must be stated before taking an expectation. *Units:* stated currency at valuation time. *In Chapter 7.1 only.*

$y_i$ **CDS event payoff value**. Constant value taken by the generic payoff on one event in a finite partition. If the payoff varies inside an event, conditional expectation replaces this constant. *Units:* stated currency at valuation time. *In Chapter 7.1 only.*

$c_i$ **CDS extinguishing swap cash flow**. Deterministic signed net cash flow scheduled at one payment date for one named perspective, in the extinguishing-swap construction that generalizes the premium leg to an arbitrary schedule. Positive means received, under the signed cash-flow convention used throughout this course. *Units:* stated currency at the scheduled payment time. *In Chapter 7.1 only.*

$PV_0^{\text{ext}}$ **CDS extinguishing swap present value**. Present value, for the named perspective, of a scheduled net cash-flow stream that stops entirely once the reference entity defaults. This is the extinguishing-swap construction generalizing the premium leg. *Units:* stated currency at valuation time. *In Chapter 7.1 only.*

$\lambda(u)$ **CDS general hazard rate**. General risk-neutral default intensity, written as a function of time rather than assumed constant. It generalizes the constant hazard rate used elsewhere in this course to a deterministic function of time; the constant case is recovered by setting it equal to one fixed value for every time. *Units:* decimal intensity per model-year. *In Chapter 7.1 only.*

$i$ **CDS period index**. Selects one default interval and its following scheduled premium date. The index is bookkeeping and is not itself a time, probability, or currency amount. *In Chapter 7.1 only.*

$$i \in \{1, \dots, n\}$$

$B_0^{\text{prot}}$ **CDS protection factor**. Present value of protection per unit CDS notional in the simplified model. It includes loss given default, discounting, and the risk-neutral default-time density. *Units:* present currency-units per notional currency-unit. *In Chapter 7.1 only.*

$A_0^{\text{sched}}$ **CDS scheduled premium annuity**. Present-value factor for scheduled premiums per unit notional and unit annual running rate. It includes a scheduled payment only when the name survives to that payment date. *Units:* model-years. *In Chapter 7.1 only.*

$r$ **Continuously compounded risk-free rate**. Deterministic annual decimal rate used to discount this lesson's model cash flows. It is a pricing input and is separate from the hazard rate. *Units:* decimal rate per model-year. *In Chapter 7.1 only.*

$$D(0, u) = \exp(-ru)$$

$n$ **Number of CDS periods**. Counts the finite premium and default intervals in the simplified CDS schedule. It is a positive integer determined by model term and payment frequency. *In Chapter 7.1 only.*

$$n \in \{1, 2, ...\}$$

$A(0, t)$ **Accumulation factor**. Grows one current unit over a stated future horizon under the selected rate model. *Units:* dimensionless currency-units per current currency-unit.

1 bp **Basis point**. Rate-change unit equal to one hundredth of one percentage point. *Units:* decimal rate.

$m_{\text{B}}$ **Bond payment frequency**. Number of scheduled coupon payments per year in the simplified regular bond, a positive integer. *Units:* scheduled coupon payments per year.

$\alpha_i$ **CDS accrual year fraction**. Input year fraction that converts an annualized spread into the premium amount for one scheduled period; a positive model input. *Units:* model-years of premium accrual.

$s$ **CDS contract spread**. Annualized premium rate applied to notional and each stated accrual year fraction; a positive rate paid by the protection buyer in the simplified lesson. *Units:* decimal per year in calculations; basis points per year when explicitly quoted.

$N$ **CDS notional**. Reference currency amount that scales the simplified premium and protection legs; a positive amount, not itself a signed leg cash flow. *Units:* stated currency.

$s^\star$ **CDS par spread**. Contractual spread that makes the two positive leg magnitudes equal at valuation time with zero upfront amount, solved under the lesson's input curves, recovery, timing, and accrued-premium convention. *Units:* decimal per year in calculations; basis points per year when explicitly quoted.

$A_0^{\text{prem}}$ **CDS premium annuity**. Present-value coefficient that multiplies contractual spread and notional in the simplified premium leg; positive, and includes scheduled premiums and exact accrued premium under the lesson's default-time model. *Units:* model-years of present value per unit notional.

$PV_0^{\text{prem}}$ **CDS premium leg present value**. Positive valuation-time magnitude of the simplified protection buyer's premium payments; its signed contribution to protection-buyer net value is negative. *Units:* stated currency at valuation time.

$PV_0^{\text{buyer}}$ **CDS protection buyer net present value**. Signed protection-buyer value equal to protection-leg magnitude minus premium-leg magnitude; positive favors the protection buyer and negative favors the protection seller in the two-leg toy model. *Units:* stated currency at valuation time.

$PV_0^{\text{prot}}$ **CDS protection leg present value**. Positive valuation-time magnitude of the simplified loss-given-default payment received by the protection buyer after a modeled default. *Units:* stated currency at valuation time.

$m$ **Compounding frequency**. Number of equal compounding periods per year under the stated rate convention, a positive integer fixed by the model convention. *Units:* compounding periods per year.

$\tau$ **Default time**. Random model time at which the reference entity first defaults. It is measured in model-years from valuation time under the stated risk-neutral model. *Units:* model-years after valuation time.

$$\tau > 0$$

$D(0,t)$ **Discount factor**. Converts one deterministic future unit into value at valuation time. *Units:* current currency-units per future currency-unit.

$\mathbb{E}$ **Expectation**. Operator returning the average of a random quantity, each outcome weighted by its probability under a stated measure; a superscript names that measure when more than one is in play. *Units:* value units.

$$\mathbb{E}[X] = \int_\Omega X(\omega)\, d\mathbb{P}(\omega)$$

$\lambda$ **Constant hazard rate**. Constant conditional default intensity used by the lesson's simplified exponential survival model; a risk-neutral pricing input conditional on survival to the current instant, not a cumulative probability. *Units:* decimal intensity per model-year.

$\Delta q_i$ **Interval default probability**. Probability assigned by the model to first default during one stated time interval, conditional only through the input survival curve construction. *Units:* probability between zero and one.

LGD **Loss given default**. Fraction of an explicitly stated reference amount not recovered under a deterministic recovery convention, relative to the same reference amount used by the recovery rate. *Units:* decimal fraction between zero and one.

$T$ **Maturity time**. Final scheduled time when principal is redeemed in the simplified bond. *Units:* model-years from the valuation date.

$j^{(m)}$ **Nominal annual rate**. Annualized rate quote that must be paired with its compounding frequency. *Units:* decimal rate per year.

$t_k$ **Payment time**. Time from the valuation date to one scheduled cash-flow event; a time coordinate, not a calendar date or payment index. *Units:* years from the valuation date.

$r_m$ **Periodic rate**. Rate applied once in each compounding period under the stated convention, derived from the stated nominal annual quote in this model. *Units:* decimal per compounding period.

$R$ **Recovery rate**. Fraction of a stated reference amount recovered after a modeled default under an explicitly stated recovery convention; a non-negative fraction whose reference amount, payment timing, and settlement convention are set by the model that uses it. *Units:* decimal fraction between zero and one.

$\mathbb{Q}$ **Risk-neutral probability measure**. Gives model pricing weights under which discounted traded prices satisfy the martingale condition; it prices payoffs relative to a stated numeraire and is not a forecast of actual event frequencies. *Units:* dimensionless probability weights between zero and one.

$S(0,t)$ **Survival probability**. Probability, under the explicitly stated model measure, that no modeled default has occurred between valuation time and a stated future time. *Units:* probability between zero and one.

$t$ **Time from valuation**. A time measured in years from the valuation time; a time coordinate, not a calendar date or a schedule index. With a row index it is a payment time. *Units:* years from the valuation date.

0 **Valuation time**. Common origin from which later model times and present values are measured. *Units:* years from the valuation date.

$s_{\text{MSQ}}$ **CDS market-standard quote**. Conventional spread that is the input or the output of the lesson's simplified converter; it equals the zero-upfront par spread within the converter's implied flat-hazard model and need not equal the contract's fixed running coupon. *Units:* decimal per year in calculations; basis points per year when explicitly quoted.

$c_{\text{std}}$ **CDS standard coupon**. Fixed annualized rate used to determine the contract's running premium cash flows in the lesson's standard-coupon model; a positive rate paid by the protection buyer on surviving notional and as accrued premium after default. *Units:* decimal per year in calculations; basis points per year when explicitly quoted.

$U_0$ **CDS upfront amount**. Time-zero cash amount that balances protection and fixed-coupon premium value under the lesson's pricing convention; positive means paid by the protection buyer and negative means received by the protection buyer. *Units:* stated currency at valuation time.

$\Pi^{\text{buyer}}$ **CDS buyer discounted cash flows**. Random sum of the protection buyer discounted signed cash flows over the contract life before any upfront: protection received on default less scheduled and accrued premium paid, each discounted from its own payment time to valuation time. *Units:* stated currency at valuation time. *In Chapter 7.3 only.*

$\lambda_k^{\text{flat}}$ **CDS flat tenor hazard rate**. Single constant hazard rate that the standard converter implies for one tenor on its own when it transforms that tenor market-standard quote into an upfront. Different tenors give different flat rates that do not form one term structure. *Units:* decimal intensity per model-year. *In Chapter 7.3 only.*

$\lambda_k$ **CDS segment hazard rate**. Constant risk-neutral hazard rate on the fitted curve segment between two consecutive tenor maturities. It is one piece of a piecewise-constant term structure, not a separate flat model for that tenor. *Units:* decimal intensity per model-year. *In Chapter 7.3 only.*

$$\lambda_k \geq 0$$

$K$ **CDS tenor count**. Counts the quoted tenors on the credit curve. It is a positive integer and does not measure time, money, or probability. *In Chapter 7.3 only.*

$$K \in \{1, 2, ...\}$$

$k$ **CDS tenor index**. Selects one quoted tenor of the credit curve in increasing maturity order. The index is bookkeeping and is not itself a time, a quote, or a probability. *In Chapter 7.3 only.*

$$k \in \{1, \dots, K\}$$

$T_k$ **CDS tenor maturity**. Maturity of the standard contract quoted at one tenor, measured in model-years from valuation time. Two consecutive tenor maturities bound one hazard segment of the fitted curve. *Units:* model-years from valuation time. *In Chapter 7.3 only.*

$s_k$ **CDS tenor quote**. Market-standard quote marked at one tenor: the conventional spread that the standard converter turns into that tenor traded upfront using a flat hazard rate for that tenor alone. *Units:* decimal per year in calculations; basis points per year when explicitly quoted. *In Chapter 7.3 only.*

$u_k$ **CDS tenor upfront fraction**. Signed upfront fraction of notional exchanged at valuation time for the standard contract at one tenor; positive means paid by the protection buyer. It is the traded price that the tenor market-standard quote stands for. *Units:* fraction of notional at valuation time. *In Chapter 7.3 only.*

$$u_k = U_0/N$$

$PV_0$ **Present value**. Combines dated signed cash flows into one value at valuation time, using the same holder perspective as the signed cash flows. *Units:* stated currency at the valuation time.

$CF_k$ **Signed cash flow**. Amount received or paid at one event from the stated holder perspective; positive means received and negative means paid by that holder. *Units:* stated currency units.

$N_{\text{eq}}$ **CDS equivalent notional**. Signed notional of the liquid contract, bought as protection, whose quote sensitivity cancels the position sensitivity; a negative value means protection is sold in the liquid tenor. *Units:* stated currency. *In Chapter 7.4 only.*

$\rho_{\text{eq}}$ **CDS equivalent ratio**. Absolute equivalent notional divided by position notional: how much of the most liquid contract offsets one unit of the position quote risk. It is positive; the side is stated separately. *In Chapter 7.4 only.*

$$\rho_{\text{eq}} = |N_{\text{eq}}|/N_{\text{pos}}$$

$q$ **CDS liquid quotation**. Any one quotation of the liquid contract viewed as a function of the flat hazard rate: its market-standard quote, its upfront fraction, or its par spread. Each is a strictly increasing function of the hazard rate in this model. *In Chapter 7.4 only.*

$T_L$ **CDS liquid tenor maturity**. Maturity of the most liquid standard contract on the reference entity, whose market-standard quote gives the single market level of the flat-hazard model. *Units:* model-years from valuation time. *In Chapter 7.4 only.*

$\text{DV01}^L$ **CDS liquid unit quote sensitivity**. Change in the value of one unit of notional of bought liquid-tenor protection when the liquid quote rises by the bump. It is positive, because bought protection gains when the quote rises. *Units:* fraction of notional per basis point. *In Chapter 7.4 only.*

$v_L$ **CDS liquid unit value**. Value at valuation time of one currency unit of notional of bought protection in the liquid contract, before upfront. It is a fraction of notional. *Units:* fraction of notional at valuation time. *In Chapter 7.4 only.*

$N_{\text{pos}}$ **CDS position notional**. Positive notional of the position being hedged, in currency units. It carries no sign; the position side does. *Units:* stated currency. *In Chapter 7.4 only.*

$\text{DV01}^{\text{pos}}$ **CDS position quote sensitivity**. Change in the signed position value when the liquid tenor market-standard quote rises by the bump, with every contract revalued at the re-solved flat hazard rate. Market usage calls this a risky DV01 or spread DV01. *Units:* stated currency per basis point. *In Chapter 7.4 only.*

$\sigma$ **CDS position side**. Sign of the position side: plus one for bought protection and minus one for sold protection. It converts protection-buyer value into value for the position holder. *In Chapter 7.4 only.*

$$\sigma \in \{+1, -1\}$$

$V^{\text{pos}}$ **CDS position value**. Signed value of the position for its own side at valuation time, before any upfront exchanged at inception: the protection-buyer value of the contract multiplied by the side sign. *Units:* stated currency at valuation time. *In Chapter 7.4 only.*

$h$ **CDS quote bump**. Size of the bump applied to the liquid tenor quote in the bump-and-reprice sensitivity. The lesson uses one basis point of market-standard quote. *Units:* decimal rate per year. *In Chapter 7.4 only.*

# Appendix B

# Abbreviations

**CDS** Credit default swap, the credit derivative the CDS lessons define and value.

**CDX** The name of a family of standard credit default swap indices, each a standard portfolio of single-name contracts.

**DTS** Duration times spread, a spread-risk measure for bonds.

**FINRA** Financial Industry Regulatory Authority.

**IMM** International Money Market. In the CDS lessons, IMM dates are the standard maturity dates on the twentieth of March, June, September, and December.

**ISDA** International Swaps and Derivatives Association.

**ISO** International Organization for Standardization.

**JTD** Jump to default, the loss on an immediate default of the reference entity.

**UTC** Coordinated Universal Time, the time standard the date arithmetic counts calendar days in.