Bonds

Part 2 of Credit Products Playground

danieltuzes

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https://danieltuzes.github.io/credit-and-derivatives/

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Part 2

Bonds

Part 1 gave a timeline of signed cash flows, rate quotes with their compounding, discount factors, and present value as a sum of discounted dated amounts. A fixed-rate bond is the first contract the course values with these tools: its contract terms fix the promised payments, and its price is their present value. The last chapter, on bond forwards, also uses the forward contract of Part 4.

This part values a simplified fixed-rate bond with deterministic promised payments. The first chapter turns face value, coupon rate, payment frequency, and maturity into the payment schedule. The next two chapters price the bond, first from one discount factor per payment and then from a single yield to maturity. The fourth chapter explains why the price decreases when the yield increases and why the price-yield curve is not linear. The last two chapters add trading details: accrued interest and the clean and dirty price at settlement, and the delivery price of a bond forward.

The argument

1. The contract terms fix every promised payment
2. The price is the present value of those payments
3. A yield to maturity summarizes the discounting in one rate
4. A higher yield gives a lower price, along a curve
5. Accrued interest separates the quoted clean price from the cash dirty price
6. Cash-and-carry gives a bond forward delivery price on the dirty-price basis

After this part you can

- Identify face value, annual coupon rate, coupon frequency, maturity, and principal redemption in a simplified fixed-rate bond description.
- Calculate each level coupon as face value times annual coupon rate divided by payments per year.
- Construct the coupon and principal cash-flow schedule of a simplified fixed-rate bond.
- Price a simplified fixed-rate bond by multiplying each promised cash flow by its stated discount factor and summing the results.
- Interpret yield to maturity as the single nominal annual rate, compounded at coupon frequency, that reproduces price from promised cash flows in the simplified model.
- Price a simplified fixed-coupon bond from a nominal annual yield compounded at the coupon frequency.
- Explain why the price of positive fixed cash flows moves inversely to yield while discounting denominators remain positive.
- Recognize that equal yield changes generally produce unequal price changes in the simplified fixed-cash-flow model.
- Calculate actual/actual coupon-period accrued interest and convert consistently between clean quoted price and dirty invoice price.

- Calculate a no-arbitrage bond forward delivery price from the current dirty bond price, the present value of pre-delivery coupons, and the delivery-date discount factor.

Builds on

- Foundations
- Derivative foundations

Leads to

- Bond options

Chapter 2.1

Fixed-rate bond contract and cash flows

Interactive version: https://danieltuzes.github.io/credit-and-derivatives/bonds/fixed-rate-contract-and-cash

2.1.1 What you will be able to do

After this lesson, you should be able to:

- identify the terms that determine a toy fixed-rate bond's payments;
- calculate the level coupon amount;
- construct the complete promised cash-flow schedule.

2.1.2 What a bond is

A bond is a debt claim. The issuer is the borrower; the bondholder is the lender who owns the claim. In the fixed-rate example used here, the issuer promises a sequence of coupon payments and repayment of principal at maturity. The price paid for the bond is a different quantity from the promised cash flows.

The word "promises" is important. A contract can specify an amount without a guarantee that the issuer pays it. This first bond lesson constructs the contractual schedule. The credit-risk lessons introduce default probability and recovery later. Tuckman and Serrat define a coupon bond by its coupon rate, its maturity, and its face amount (also called the par amount or the principal amount). They show the coupons and the principal on the cash-flow timeline[1].

2.1.3 Read the contract before valuing it

Write the simplified contract's face-value as F, its annual-coupon-rate as c, its bond-payment-frequency as m_{B}, and its maturity-time as T. Write the number-of-bond-payments as n and index those dates with the bond-payment-index k. The matching payment-time is t_k, measured from valuation-time 0.

Write the level coupon-payment as C and the promised bond-cash-flow on date k as CF_k^{bond}.

The number of payment dates is the payment frequency multiplied by the maturity:

$$n = m_{\mathrm{B}} T \tag{2.1.1}$$

The bond payment frequency is a different concept from the compounding-frequency m of the earlier lessons. So it has a different symbol, with the subscript B.

2.1.4 Calculate the coupon

The level coupon payment is the annual coupon rate multiplied by the face value, divided by the payment frequency:

$$C = \frac{cF}{m_{\mathrm{B}}} \tag{2.1.2}$$

The coupon rate is applied to the face value, not to the market price of the bond.

2.1.5 Construct every promised payment

For the bondholder, the promised bond cash flow on date k is the coupon, plus the face value on the final date:

$$CF_k^{\mathrm{bond}} = C + \mathbf{1}_{\{k=n\}} F \tag{2.1.3}$$

The indicator $\mathbf{1}_{\{k=n\}}$ equals one on the final date and zero on every other date. So the face value is paid only on the final date.

On the regular schedule, the payment time of date k, measured from valuation time, is:

$$t_k = \frac{k}{m_{\mathrm{B}}}, \qquad 0 \text{ is the valuation time} \tag{2.1.4}$$

Example 2.1.1: Coupon schedules by payment frequency

Annual coupons

A two-year bond has a face value of USD 1,000, an annual coupon rate of 5%, and one payment per year. The coupon is:

$$C = \frac{0.05 \times 1000}{1} = \text{USD } 50$$

Payment index k	Payment time t_k, years	Coupon, USD	Principal, USD	Total, USD
1	1.0	50	0	50
2	2.0	50	1,000	1,050

Semiannual coupons

An 18-month bond has a face value of USD 1,000, an annual coupon rate of 4%, and two payments per year. The coupon and the number of payments are:

$$C = \frac{0.04 \times 1000}{2} = \text{USD } 20, \qquad n = 2 \times 1.5 = 3$$

Its payments at 0.5, 1.0, and 1.5 years are USD 20, USD 20, and USD 1,020.

Quarterly coupon amount

For a face value of USD 500, an annual coupon rate of 4.8%, and quarterly payments, the coupon is:

$$C = \frac{0.048 \times 500}{4} = \text{USD } 6$$

The maturity determines the number of payments. Under the level-coupon assumption, the maturity does not change the coupon amount.

2.1.6 Check your understanding

These items cover term interpretation, coupon arithmetic, and schedule construction. Each question stays collapsed until you open it; answers and explanations appear once you check.

Knowledge check 2.1.1: Fixed-rate bond contract and cash flows

1. A simplified bond has USD 1,000 face value, a 5% annual coupon rate, two coupon payments per year, and two years to maturity. Which field determines the principal redeemed at maturity under the lesson convention?

   (a) Face value
   (b) Annual coupon rate
   (c) Coupon frequency

2. A simplified fixed-rate bond's market price changes while its contract is unchanged. Which listed field remains a contractual input to promised coupon cash flows?

   (a) The annual coupon rate
   (b) The new market price
   (c) The size of the market-price change

3. Under the lesson convention, a bond has USD 1,000 face value, a 6% annual coupon rate, and two equal coupon payments per year. What is each coupon amount in USD?

4. Under the lesson convention, a bond has USD 2,000 face value, a 4.5% annual coupon rate, and two equal coupon payments per year. What is each coupon amount in USD?

5. From the holder's perspective, what are the promised cash flows of a two-year bond with USD 1,000 face value, a 5% annual coupon rate, and one coupon payment per year?

   (a) USD +50 after year one and USD +1,050 after year two
   (b) USD +50 after each year and no principal
   (c) USD +1,100 after year two only

6. From the holder's perspective, what are the promised cash flows of an 18-month bond with USD 1,000 face value, a 4% annual coupon rate, and semiannual coupons?

(a) USD +20, USD +20, and USD +1,020 at six-month intervals

(b) USD +40, USD +40, and USD +1,040 at six-month intervals

(c) USD +1,020 at every coupon date

Answers: *end of the chapter.*

2.1.7 Model boundary and review note

This lesson describes promised cash flows only. It does not assign a price, yield, payment probability, or expected recovery.

The contract terms and the level-coupon schedule follow Tuckman & Serrat[2]. The lesson stays `draft` pending human confirmation of the printed locators.

References

[1] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 1 §1.1, printed p. 50 and Table 1.1.

[2] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). §1.1, "Government Coupon Bonds": a bond is set by coupon rate, maturity date, and face (par, principal); each semiannual coupon is half the annual rate times face, with principal repaid at maturity (Table 1.1).

Answers to the knowledge checks

Knowledge check 2.1.1: Fixed-rate bond contract and cash flows

1. **(a)** Face value. The simplified contract redeems principal at face value; the coupon rate and frequency determine periodic coupons.

2. **(a)** The annual coupon rate. In the simplified model, promised coupons follow the contractual face value, annual coupon rate, and frequency rather than the current market price.

3. 30 (accepted within $\pm 1e-9$). Coupon = face value $\times$ annual coupon rate / payments per year = 1,000 $\times$ 0.06 / 2 = 30.

4. 45 (accepted within $\pm 1e-9$). Coupon = 2,000 $\times$ 0.045 / 2 = 45.

5. **(a)** USD +50 after year one and USD +1,050 after year two. The coupon is 5% of face value each year; principal is added to the final coupon.

6. **(a)** USD +20, USD +20, and USD +1,020 at six-month intervals. Each semiannual coupon is 1,000 $\times$ 4% / 2 = 20, with principal paid only at maturity.

Chapter 2.2

Bond price from discount factors

Interactive version: https://danieltuzes.github.io/credit-and-derivatives/bonds/price-from-discount-factors/

2.2.1 What you will be able to do

After this lesson, you should be able to calculate a toy bond's price from its promised cash-flow schedule and input discount factors.

2.2.2 Bond pricing is a present-value application

Use the bond-payment-index k to select one of the number-of-bond-payments n. Write the promised bond-cash-flow as CF_k^{bond}, its payment-time as t_k, and its matching discount-factor as $D(0, t_k)$. Write the bond-price as P_0. The bond price is the present-value at valuation-time 0 of the promised payments. So the bond price is the schedule sum (1.5.1), applied to the promised payments of a bond:

$$P_0 = \sum_{k=1}^{n} CF_k^{\text{bond}} D(0, t_k) \tag{2.2.1}$$

In this lesson, "price" means this present value. It does not mean a clean price, a quoted price, or a probability. Those concepts are outside this lesson.

Example 2.2.1: Bond prices from discount factors

Two discount factors

The annual-coupon bond from the preceding lesson pays USD 50 after one year and USD 1,050 after two years. Suppose the matching discount factors are 0.96 and 0.91.

Payment index k	Cash flow, USD	Discount factor	Present value, USD
1	50	0.96	48.00
2	1,050	0.91	955.50

The bond price is the sum of the present values:

$$P_0 = 50(0.96) + 1050(0.91) = \text{USD } 1{,}003.50$$

The price is above USD 1,000 because, with these two discount factors, the present value of the promised payments is larger than the face value. This lesson does not use a yield.

Unfamiliar schedule

Consider payments of USD 20 at 0.5 years, USD 20 at 1.0 year, and USD 1,020 at 1.5 years, with discount factors 0.98, 0.95, and 0.92. The bond price is:

$$P_0 = 20(0.98) + 20(0.95) + 1020(0.92) = 19.60 + 19.00 + 938.40 = \text{USD } 977.00$$

Zero-coupon boundary

If every coupon is zero, the only payment is the face-value F at maturity-time T. The bond price is then:

$$P_0 = FD(0, T)$$

This is (2.2.1) with one payment. It is not a separate rule.

2.2.3 Error pattern: discount after adding

For the two-payment example, compare a wrong calculation, which applies one discount factor to the sum of the payments, with the calculation in (2.2.1):

$$\begin{aligned} (50 + 1050)(0.91) &= 1001.00 \\ 50(0.96) + 1050(0.91) &= 1003.50 \\ 1001.00 &\neq 1003.50 \end{aligned} \tag{2.2.2}$$

One common discount factor is correct only when the model assigns that discount factor to every payment.

2.2.4 Check your understanding

This set includes one coupon-bond calculation and one transfer schedule. Each question stays collapsed until you open it; answers and explanations appear once you check.

Knowledge check 2.2.1: Bond price from discount factors

1. From the holder's perspective, a simplified bond pays USD +50 at t = 1 year and USD +1,050 at t = 2 years. The matching discount factors are 0.95 and 0.90. What is its time-zero price in USD?

2. From the holder's perspective, a simplified bond pays USD +20, USD +20, and USD +1,020 at three successive dates. Their matching discount factors are 0.98, 0.95, and 0.91. What is its time-zero price in USD?

Answers: end of the chapter.

2.2.5 Model boundary and review note

The discount factors are inputs; they are not calibrated. This lesson has no credit risk, although the course is about credit products.

The pricing rule (2.2.1) follows Tuckman & Serrat[1]; Hull discounts each cash flow at its own zero rate the same way[2]. The lesson stays `draft` pending human confirmation of the printed locators.

References

[1] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). §1.2, each bond price as its cash flows times the matching discount factors (eqs. 1.1-1.3), checked against market prices in Table 1.4.

[2] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 4, "Bond Pricing".

Answers to the knowledge checks

Knowledge check 2.2.1: Bond price from discount factors

1. 992.5 (accepted within $\pm 1e-9$). Price $= 50 \times 0.95 + 1{,}050 \times 0.90 = 47.5 + 945 = 992.5$.
2. 966.8 (accepted within $\pm 1e-9$). Price $= 20 \times 0.98 + 20 \times 0.95 + 1{,}020 \times 0.91 = 19.6 + 19 + 928.2 = 966.8$.

Chapter 2.3

Yield to maturity as a single-rate summary

Interactive version: https://danieltuzes.github.io/credit-and-derivatives/bonds/yield-to-maturity/

2.3.1 What you will be able to do

After this lesson, you should be able to:

- interpret yield to maturity as a model-dependent single-rate summary;
- state its annualization and compounding convention;
- calculate the price from an input yield without changing the promised cash flows.

2.3.2 What yield means in this toy model

Use the bond-payment-index k to select one of the number-of-bond-payments n, and write its promised bond-cash-flow as CF_k^{bond}. Write the bond-price as P_0 and the bond-payment-frequency as m_{B}.

Write yield-to-maturity as $y^{(m_{\text{B}})}$. It is the one nominal annual rate, compounded at the bond payment frequency, that makes the discounted promised payments equal the bond price:

$$P_0 = \sum_{k=1}^{n} \frac{CF_k^{\text{bond}}}{\left(1 + y^{(m_{\text{B}})}/m_{\text{B}}\right)^k} \tag{2.3.1}$$

The symbol $y^{(m_{\text{B}})}$ could be mistaken for a spot rate, an effective annual rate, an expected return, or another quote. In this course it means only the bond yield defined above.

2.3.3 Coupon rate and yield have different roles

- The coupon rate determines the promised coupon-payment C in the contract.
- The input yield determines the denominators in the price formula.

A change of the input yield does not change the coupons or the face-value F of the bond.

Example 2.3.1: Pricing from a yield to maturity

Annual payments

A two-year bond with an annual coupon rate of 5% and a face value of USD 1,000 pays USD 50 and USD 1,050. At an annual yield of 6% with annual compounding, the price is:

$$P_0 = \frac{50}{1.06} + \frac{1050}{1.06^2} \approx \text{USD } 981.67$$

Semiannual compounding

For the same face value and coupon rate over two years, but with semiannual payments, $C = 25$, $m_{\mathrm{B}} = 2$, and $n = 4$. A nominal annual yield of 6% gives a periodic rate of 3%, and the price is:

$$P_0 = \frac{25}{1.03} + \frac{25}{1.03^2} + \frac{25}{1.03^3} + \frac{1025}{1.03^4} \approx \text{USD } 981.41$$

The annual and the semiannual example use the same yield number, 6%, but different payment times and different compounding.

Zero-coupon special case

With no coupons and maturity-time T, the price is:

$$P_0 = \frac{F}{\left(1 + y^{(m_{\mathrm{B}})}/m_{\mathrm{B}}\right)^{m_{\mathrm{B}}T}}$$

For USD 1,000 due in three years at 4% with annual compounding, the price is approximately USD 889.00.

2.3.4 Check your understanding

These items separate yield interpretation from annual and periodic-compounding calculations. Each question stays collapsed until you open it; answers and explanations appear once you check.

Knowledge check 2.3.1: Yield to maturity

1. In the simplified lesson model, what is yield to maturity?
   (a) The single nominal annual rate, compounded at coupon frequency, that reproduces price from promised cash flows
   (b) An additional cash flow paid by the issuer at maturity
   (c) The currency amount of each coupon
2. Two bonds have different promised cash-flow amounts and maturities, but the same coupon frequency and the same quoted yield to maturity under the simplified nominal convention. Which statement follows?
   (a) The same summary rate is used, but each bond's own cash flows still determine its price
   (b) The two bonds must have the same price
   (c) The two bonds must have identical coupon cash flows

3. Price in USD a two-year bond with USD 1,000 face value, a 5% annual coupon rate, and one coupon payment per year at a 6% nominal annual yield compounded annually. Assume valuation on a coupon date and no accrued interest.

4. Price in USD a two-year bond with USD 1,000 face value, a 5% annual coupon rate, and two coupon payments per year at a 6% nominal annual yield compounded semiannually. Assume valuation on a coupon date and no accrued interest.

Answers: *end of the chapter.*

2.3.5 Model boundary and review note

This lesson calculates the price from an input yield. It does not solve numerically for the yield from a price. A real bond implementation also needs schedules, day counts, accrued interest, clean and dirty prices, and possibly credit and option models.

The single-rate definition and the $2T$-period pricing formula follow Tuckman & Serrat[1]; Hull gives the same definition, solved iteratively[2]. FINRA defines it as the return to an investor who buys at the market price and holds to maturity[3]. The lesson stays `draft` pending human confirmation of the printed locators.

References

[1] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). §3.2, "Yield to Maturity": the single rate that discounts a bond's cash flows to its market price (eq. 3.5 and the general formula), also its internal rate of return.

[2] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 4, "Bond Yield".

[3] FINRA, Understanding Bond Yield and Return. "Yield to maturity" section. https://www.finra.org/investors/insights/bond-yield-return

Answers to the knowledge checks

Knowledge check 2.3.1: Yield to maturity

1. **(a)** The single nominal annual rate, compounded at coupon frequency, that reproduces price from promised cash flows. The toy model defines yield to maturity as one rate that discounts all promised cash flows to the stated price under the stated compounding convention.

2. **(a)** The same summary rate is used, but each bond's own cash flows still determine its price. A common yield does not erase differences in amount or timing; the model discounts each bond's own promised cash flows.

3. 981.6660733357066 (accepted within ± 0.01). $P_0 = \frac{50}{1.06} + \frac{1{,}050}{1.06^2}$.

4. 981.4145079859479 (accepted within ± 0.01). Each coupon is USD 25 and the periodic yield is $0.06/2 = 0.03$, so $P_0 = \frac{25}{1.03} + \frac{25}{1.03^2} + \frac{25}{1.03^3} + \frac{1{,}025}{1.03^4}$.

Chapter 2.4

The bond price-yield relationship

Interactive version: https://danieltuzes.github.io/credit-and-derivatives/bonds/price-yield-relationship/

2.4.1 What you will be able to do

After this lesson, you should be able to:

- explain why the price of a bond with fixed positive cash flows decreases when the yield increases;
- hold contract cash flows fixed while changing only the discounting input;
- recognize that the price-yield relationship is curved rather than linear.

2.4.2 The price decreases because every denominator increases

Write the bond-payment-frequency as m_{B}. In this lesson, y denotes the yield-to-maturity $y^{(m_{\mathrm{B}})}$ without its superscript, as the argument of a function. Write the price-yield-curve as $P(y)$.

Use the bond-payment-index k to select one of the number-of-bond-payments n, and write each promised positive bond-cash-flow as CF_k^{bond}. The price at yield y is the sum of the promised cash flows, each discounted at the yield:

$$P(y) = \sum_{k=1}^{n} \frac{CF_k^{\mathrm{bond}}}{\left(1 + y/m_{\mathrm{B}}\right)^k} \tag{2.4.1}$$

Assume that the yield stays non-negative and that every cash flow stays fixed. If the yield increases, every denominator increases. So every discounted cash flow decreases, and the bond-price decreases. Table 2.4.1 also assumes that the new yield is non-negative.

Define the local yield-shock Δy as new yield minus old yield, and define the local price-change ΔP as new price minus old price.

The table holds only if the promised cash flows stay fixed.

Example 2.4.1: Price-yield comparisons

Three points form a curve

Consider a five-year bond with an annual coupon rate of 5%, semiannual payments, and a face value of 100. Its prices at three yields are:

Table 2.4.1 **Signs of a yield shock and the price change.** For a rise and a fall in the yield, the direction of every discount denominator and the sign of the price change, with the promised cash flows held fixed.

Yield shock	Denominators	Price change
$\Delta y > 0$	Increase	$\Delta P < 0$
$\Delta y < 0$	Decrease	$\Delta P > 0$

Nominal annual yield	Price per 100 face
5%	100.0000
6%	95.7349
7%	91.6834

The price decreases as the yield increases. Each yield increase is one percentage point, but the two price changes are different: 4.2651 and 4.0515. So the three points are not on a straight line.

Equal shocks, unequal price changes

Start from a yield of 5%:

- a yield decrease to 4% increases the price from 100.0000 to 104.4913, a change of +4.4913;
- a yield increase to 6% decreases the price from 100.0000 to 95.7349, a change of −4.2651.

The two yield shocks have equal size, but the absolute price changes are different. This difference is a result of the curvature of the price-yield curve. This lesson does not calculate duration, DV01, or convexity.

Hold other inputs fixed

Suppose the issuer changes a coupon, a default occurs, or the settlement date changes. Then more than the yield has changed, and the comparison no longer isolates the effect of the yield.

One tested domain function calculates every price in Figure 2.4.1. The notation panel only explains the symbols; it does not calculate prices.

2.4.3 Check your understanding

These direct and transfer items cover direction and nonlinearity. Each question stays collapsed until you open it; answers and explanations appear once you check.

Knowledge check 2.4.1: Bond price-yield shape

1. Within the lesson's non-negative-yield domain, all else equal, what happens to a fixed bond's price when its yield rises?

   (a) The price falls

   (b) The price rises

   (c) The price is unchanged

2. A bond's yield falls from 6% to 5% while promised cash flows stay fixed. Which explanation

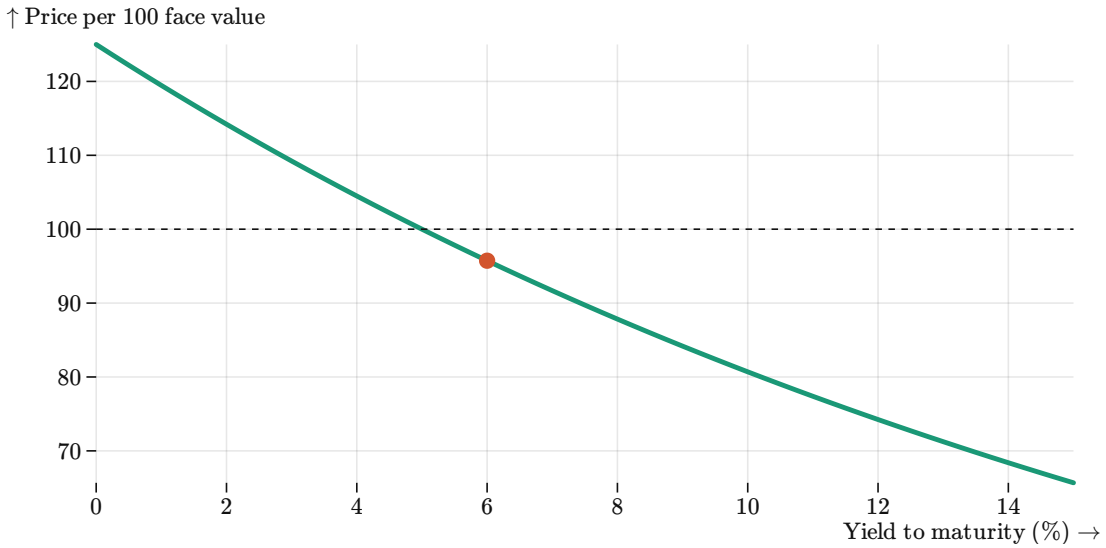


Figure 2.4.1 **Bond price–yield explorer.** The price per 100 face value of one fixed-coupon bond at every yield to maturity from 0% to 15%. The dot marks the price at the yield shown; the dashed line marks par, a price of 100. Shown for a 5% annual coupon rate, 2 coupon payments a year and a 5-year maturity: at a 6% yield to maturity, the price is 95.73 per 100 face value.

is correct?

(a) Discounting denominators shrink, so price rises

(b) The contractual coupons grow, so price rises

(c) The principal shrinks, so price falls

3. A fixed bond's prices at yields of 4%, 5%, and 6% are 103.0, 100.0, and 97.2. What do the unequal changes for equal one-percentage-point yield steps show?

(a) The price-yield relationship is not a straight line

(b) Price is independent of yield

(c) The promised cash flows must change with yield

4. For three equally spaced yields, the middle bond price differs from the average of the two endpoint prices. What is the best interpretation?

(a) The sampled price-yield relationship has curvature

(b) The sampled price-yield relationship is exactly linear

(c) Yield has no relationship to price

Answers: *end of the chapter.*

2.4.4 Model boundary and review note

The plotted curve summarizes a toy model with one yield. It is not a credit curve and not a recommendation. It omits accrued interest, term-structure effects, default, recovery, liquidity, tax,

funding, and embedded options.

The inverse direction and the curvature both follow Tuckman & Serrat[1]; Hull notes the same negative price-yield relationship[2]. FINRA states that price and yield are inversely related[3]. The lesson stays `draft` pending human confirmation of the printed locators.

References

[1] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). §4.1, "Price-Rate Curves", Figure 4.2, prices decrease as rates rise; §4.5, "Convexity", price-rate curves of coupon bonds are convex.

[2] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). §4.8, Duration.

[3] FINRA, Understanding Bond Yield and Return. "Price and yield" section. https://www.finra.org/investors/insights/bond-yield-return

Answers to the knowledge checks

Knowledge check 2.4.1: Bond price-yield shape

1. **(a)** The price falls. A higher yield produces smaller discount factors for every positive future cash flow.

2. **(a)** Discounting denominators shrink, so price rises. The promised coupons do not change; the change comes from discounting them at a lower yield.

3. **(a)** The price-yield relationship is not a straight line. The first equal yield step changes price by 3.0 while the second changes it by 2.8, so the slope is not constant.

4. **(a)** The sampled price-yield relationship has curvature. For a straight line, the value at the midpoint equals the average of endpoint values; a difference is evidence of curvature.

Chapter 2.5

Settlement, accrued interest, and clean versus dirty price

Interactive version: https://danieltuzes.github.io/credit-and-derivatives/bonds/settlement-clean-and-dirty-p

2.5.1 The price paid is not always the price quoted

A bond price quote commonly shows the clean-bond-price, also called the flat or quoted price. The buyer pays the dirty-bond-price, also called the full, cash, or invoice price. The dirty price is the clean price plus the accrued-interest:

$$P^{\text{dirty}} = P^{\text{clean}} + AI \tag{2.5.1}$$

On a coupon date, the accrued interest is zero, so the two prices are equal. For this reason, the earlier bond lessons, which settle on a coupon date, use one bond-price symbol.

2.5.2 Actual/actual within one coupon period

Under the convention of this lesson, let the local coupon-period-elapsed-days be n_{elapsed} and the local coupon-period-total-days be n_{period}. For one coupon-payment C, the accrued interest is the coupon multiplied by the fraction of the coupon period that has elapsed:

$$AI = C\frac{n_{\text{elapsed}}}{n_{\text{period}}} \tag{2.5.2}$$

Consider the coupon period from February 15, 2021 to August 15, 2021, which has 181 days. Settlement on May 17, 2021 is 91 actual days after the start of the period. With a coupon of USD 31.25, (2.5.2) gives the accrued interest:

$$AI = 31.25\left(\frac{91}{181}\right) = 15.71132597 \text{ USD per bond} \tag{2.5.3}$$

The domain code checks that the dates are valid ISO dates and counts UTC calendar days. It is not a production schedule engine: business-day rules, ex-coupon periods, stubs, and other day counts require separate conventions.

2.5.3 Why this matters for derivatives

The cash amount paid at the settlement of a bond forward or a bond option depends on whether its price is clean or dirty. If a strike is quoted as a clean price and the modeled bond price is a dirty price, add the accrued interest at the exercise date to the strike before the payoff is calculated. Also, each coupon paid before delivery must be assigned to the party that owns the bond on the coupon date.

Knowledge check 2.5.1: Bond settlement

1. A coupon payment is USD 31.25. Settlement is 91 actual days into an actual 181-day coupon period. Calculate accrued interest in USD.

2. A bond has a clean cash price of USD 9,178.125 and accrued interest of USD 15.711. Calculate its dirty invoice price in USD.

Answers: end of the chapter.

2.5.4 Sources

The identity between the clean and the dirty price (2.5.1) and the actual/actual calculation (2.5.2) follow Tuckman and Serrat, Chapter 1 §1.6 [1]. Hull's European bond option discussion emphasizes matching cash versus quoted strike and bond prices [2].

References

[1] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022).

[2] Hull, Options, Futures, and Other Derivatives (8th ed., 2012).

Answers to the knowledge checks

Knowledge check 2.5.1: Bond settlement

1. 15.71132596685083 (accepted within ± 0.000001). Accrued interest is $31.25 \times 91 / 181 =$ USD 15.71132597.

2. 9193.836 (accepted within ± 0.000001). Dirty price = clean price + accrued interest = USD 9,193.836.

Chapter 2.6

Bond forwards

Interactive version: https://danieltuzes.github.io/credit-and-derivatives/bonds/bond-forwards/

2.6.1 The forward contract

The holder of a long bond forward must pay the forward-delivery-price and receive the named bond at forward-delivery-time. The holder of the forward does not own the bond before delivery. So coupons paid before delivery belong to the current owner of the bond.

2.6.2 Cash-and-carry price

At valuation-time, start with the current dirty-bond-price P^{dirty}. Subtract the local predelivery-coupon-present-value I_0^{pre}. The difference is the prepaid forward value: the value today of receiving the bond at delivery. Dividing the prepaid forward value by the discount factor to delivery converts it to an amount at delivery. This amount is the fair bond-forward-price:

$$F^B_{0,T_{\text{fwd}}} = \frac{P^{\text{dirty}} - I_0^{\text{pre}}}{D(0, T_{\text{fwd}})} \tag{2.6.1}$$

For a dirty price of USD 960, a present value of the predelivery coupons of USD 95.45, and a discount factor to delivery of 0.92, (2.6.1) gives the bond forward price:

$$F^B_{0,T_{\text{fwd}}} = \frac{960 - 95.45}{0.92} = 939.72826087 \text{ USD per bond} \tag{2.6.2}$$

An existing contract can have a different delivery price. The earlier lesson on forward value calculates the value of such a contract. This lesson calculates the fair delivery price of a new contract.

2.6.3 Basis consistency

A clean price quote must first be converted to a dirty price. In the same way, the accrued interest at the delivery date must be added to a clean delivery price or a clean strike. If clean and dirty prices are mixed, the result can look like a pricing discrepancy, but it is only a mismatch of conventions.

Knowledge check 2.6.1: Bond forward

1. A bond's current dirty price is USD 960. The present value of coupons paid before delivery is USD 95.45, and D(0,T) = 0.92. Calculate the fair dirty bond forward delivery price in USD.

2. A bond's dirty spot price is USD 1,025, pre-delivery coupons have present value USD 24, and D(0,T) = 0.97. Calculate the fair dirty delivery price in USD.

Answers: end of the chapter.

2.6.4 Sources

The known-income cash-and-carry relation (2.6.1) and its coupon-bond application follow Hull, Chapter 5 §§5.3–5.5; Hull also treats the dirty-price basis of the bond forward and the strike in Chapter 28 §28.1 [1].

References

[1] Hull, Options, Futures, and Other Derivatives (8th ed., 2012).

Answers to the knowledge checks

Knowledge check 2.6.1: Bond forward

1. 939.7282608695652 (accepted within ±0.000001). The prepaid forward is 960 - 95.45 = 864.55. Divide by 0.92 to obtain USD 939.72826087.

2. 1031.958762886598 (accepted within ±0.000001). The fair delivery price is (1,025 - 24) / 0.97 = USD 1,031.95876289.

Appendix A

Notation

The symbols of this book, in the order they first appear.

$\mathbf{1}_{\{k=n\}}$ **Final payment indicator**. Equals one on the final payment date, where the payment index equals the number of payments, and zero on every other date. It places the face value on the final date only. *In Chapter 2.1 only.*

c **Annual coupon rate**. Contractual annual rate used to determine a fixed-rate bond's coupon payments; applied to face value, not to market price. *Units:* decimal rate per year.

CF_k^{bond} **Bond cash flow**. Promised amount paid to the bondholder on one scheduled payment date; a positive receipt for the bondholder, with default excluded in the bond lessons. *Units:* stated currency at payment time.

m_{B} **Bond payment frequency**. Number of scheduled coupon payments per year in the simplified regular bond, a positive integer. *Units:* scheduled coupon payments per year.

k **Bond payment index**. Labels one remaining scheduled bond payment in increasing time order; it selects a payment and is not itself a time or currency amount. *Units:* dimensionless schedule index.

m **Compounding frequency**. Number of equal compounding periods per year under the stated rate convention, a positive integer fixed by the model convention. *Units:* compounding periods per year.

C **Coupon payment**. Level periodic cash amount promised by the simplified fixed-rate bond; a positive receipt for the bondholder in these lessons. *Units:* stated currency per coupon date.

F **Face value**. Contractual reference amount used to determine coupons and principal redemption. *Units:* stated currency.

T **Maturity time**. Final scheduled time when principal is redeemed in the simplified bond. *Units:* model-years from the valuation date.

n **Number of bond payments**. Counts the remaining regular coupon dates including maturity; a positive integer for the simplified regular bond schedule. *Units:* scheduled payment dates.

t_k **Payment time**. Time from the valuation date to one scheduled cash-flow event; a time coordinate, not a calendar date or payment index. *Units:* years from the valuation date.

0 **Valuation time**. Common origin from which later model times and present values are measured. *Units:* years from the valuation date.

$A(0,t)$ **Accumulation factor**. Grows one current unit over a stated future horizon under the selected rate model. *Units:* dimensionless currency-units per current currency-unit.

P_0 **Bond price**. Present value of the simplified bond's promised payments at valuation time; the amount paid by the buyer, shown as a positive value in the bond lessons. *Units:* stated currency at valuation time.

$D(0,t)$ **Discount factor**. Converts one deterministic future unit into value at valuation time. *Units:* current currency-units per future currency-unit.

$j^{(m)}$ **Nominal annual rate**. Annualized rate quote that must be paired with its compounding frequency. *Units:* decimal rate per year.

r_m **Periodic rate**. Rate applied once in each compounding period under the stated convention, derived from the stated nominal annual quote in this model. *Units:* decimal per compounding period.

PV_0 **Present value**. Combines dated signed cash flows into one value at valuation time, using the same holder perspective as the signed cash flows. *Units:* stated currency at the valuation time.

CF_k **Signed cash flow**. Amount received or paid at one event from the stated holder perspective; positive means received and negative means paid by that holder. *Units:* stated currency units.

$y^{(m_{\mathrm{B}})}$ **Yield to maturity**. Single nominal annual rate that reproduces the simplified bond price. *Units:* nominal annual decimal rate compounded at the bond payment frequency.

ΔP **Price change**. New toy-model price minus old toy-model price for a stated yield shock. This finite change is not yet a duration or convexity approximation. *Units:* stated currency per bond at valuation time. *In Chapter 2.4 only.*

$$\Delta P = P(y + \Delta y) - P(y)$$

Δy **Yield shock**. A lesson-local finite change in the input yield. The sign is new yield minus old yield; the unit must be decimal, percentage points, or basis points as explicitly labeled. *Units:* decimal rate per year. *In Chapter 2.4 only.*

$$\Delta y = y_{\mathrm{new}} - y_{\mathrm{old}}$$

$P(y)$ **Price-yield curve**. Bond price as a function of yield while promised positive fixed cash flows remain constant and only the yield varies. *Units:* stated currency per bond.

n_{elapsed} **Coupon period elapsed days**. Actual calendar days from the previous coupon date through settlement under the lesson convention. *Units:* days. *In Chapter 2.5 only.*

n_{period} **Coupon period total days**. Actual calendar days from the previous coupon date to the next coupon date. *Units:* days. *In Chapter 2.5 only.*

AI **Accrued interest**. Coupon amount attributed to the interval from the previous coupon date through settlement under the stated day-count convention; a positive amount added to the clean price to obtain the dirty invoice price. *Units:* stated currency at settlement.

P^{clean} **Clean bond price**. Bond price quoted without accrued interest under the stated settlement convention; a positive quoted price to the buyer before accrued interest is added. *Units:* stated currency at settlement.

P^{dirty} **Dirty bond price**. Full cash or invoice price paid for the bond, equal to clean price plus accrued interest; a positive cash price paid by the buyer under the stated settlement convention. *Units:* stated currency at settlement.

I_0^{pre} **Predelivery coupon present value**. Valuation-time present value of coupons paid to the current bond owner before forward delivery. *Units:* stated currency at valuation time per bond. *In Chapter 2.6 only.*

$F^B_{0,T_{\text{fwd}}}$ **Bond forward price**. Fair dirty delivery price fixed at valuation time for delivery of the named bond at the forward date; positive delivery cash paid by the long under the lesson's no-arbitrage assumptions. *Units:* stated currency at forward delivery per bond.

K_{fwd} **Forward delivery price**. Contractual currency amount per unit of underlying paid by the long at delivery and received by the short. *Units:* delivery-time currency per unit of underlying.

T_{fwd} **Forward delivery time**. Future model time when the forward counterparties exchange the underlying and delivery payment; a contract date shared by the long and short, not the underlying asset's maturity. *Units:* model-years from the stated valuation time.

Appendix B

Abbreviations

CDS Credit default swap, the credit derivative the CDS lessons define and value.

CDX The name of a family of standard credit default swap indices, each a standard portfolio of single-name contracts.

DTS Duration times spread, a spread-risk measure for bonds.

FINRA Financial Industry Regulatory Authority.

IMM International Money Market. In the CDS lessons, IMM dates are the standard maturity dates on the twentieth of March, June, September, and December.

ISDA International Swaps and Derivatives Association.

ISO International Organization for Standardization.

JTD Jump to default, the loss on an immediate default of the reference entity.

UTC Coordinated Universal Time, the time standard the date arithmetic counts calendar days in.