Bond options

Part 5 of Credit Products Playground

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https://danieltuzes.github.io/credit-and-derivatives/

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Part 5

Bond options

Part 2 put a bond's quoted and cash prices on one basis, and Part 4 valued a European option by backward induction on a lattice. This part combines them: the underlying of the option is a bond, and the lattice gives the bond values at expiry. The second chapter also uses the survival probability and the recovery of Part 6, because issuer default ends the option.

This part values European options whose underlying is a bond. The first chapter defines a standalone call or put on a bond, puts the strike and the bond value on one cash-price basis, and values the expiry payoffs on an input lattice. The second chapter adds issuer default. Default before exercise extinguishes the option with no rebate, the bond's recovery stays a separate cash flow of the bond, and each backward step discounts only the survival-weighted option values.

The argument

1. A bond option's strike and bond value share one cash-price basis
2. The lattice gives the bond value at each expiry node
3. Backward induction values the expiry payoffs
4. Issuer default before exercise extinguishes the option
5. Each backward step weights the option value by survival

After this part you can

- Interpret a standalone European call or put on a named bond, including exercise time, dirty-versus-clean strike basis, coupon timing, and holder rights.
- Calculate a European bond option payoff at expiry and value it by backward induction on an input default-free bond-value lattice.
- Interpret an issuer-default knockout bond option as a contract extinguished with zero option rebate if the bond issuer defaults before exercise, separately from any recovery on the bond.
- Calculate an issuer-default knockout bond option value on an input joint rate-credit lattice by applying survival to option continuation and zero option payoff on pre-expiry default.

Builds on

- Bonds
- Derivative foundations
- Credit risk

Chapter 5.1

European bond options

Interactive version: https://danieltuzes.github.io/credit-and-derivatives/bond-options/european-bond-options

5.1.1 The standalone contract

A European bond call gives the holder the right, but not the obligation, to buy a named bond at option-expiry-time. A put gives the right to sell. This option is not the issuer's call feature inside a callable bond. The two rights are held by different parties and belong to different instruments.

5.1.2 Put the bond value and the strike on one price basis

The local bond-option-expiry-value $B^{\text{dirty}}_{T_{\text{opt}}}$ and option-strike-price K must both be dirty cash prices. If a contract states a clean strike, add the accrued-interest at the exercise date to the strike before the comparison.

In this lesson, exercise occurs immediately after any coupon paid on the expiry date. The call payoff and the put payoff are:

$$X^{B,\text{call}}_{T_{\text{opt}}} = \max\left(B^{\text{dirty}}_{T_{\text{opt}}} - K, 0\right) \tag{5.1.1}$$

$$X^{B,\text{put}}_{T_{\text{opt}}} = \max\left(K - B^{\text{dirty}}_{T_{\text{opt}}}, 0\right) \tag{5.1.2}$$

The lattice first gives the bond values at expiry. Apply (5.1.1) or (5.1.2) at the expiry nodes. Then apply backward induction to calculate the lattice-node-value at time zero.

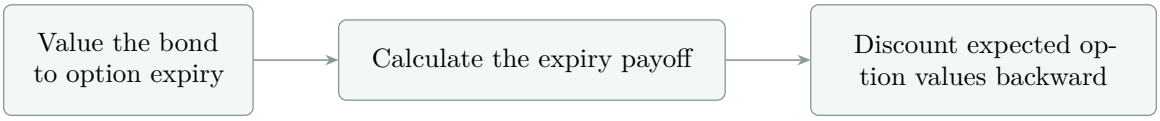


Diagram 5.1.1 **European bond-option valuation flow.** First value the bond at every option-expiry node, then apply the call or put payoff, then discount the risk-neutral expected payoffs backward through the input lattice.

5.1.3 Worked example

Suppose a one-period input lattice gives dirty bond values at expiry of USD 90 and USD 110. The strike is USD 100, the risk-neutral up weight is 0.60, and the discount factor is 0.95. Applying (5.1.1) and one backward step gives the call value:

$$\begin{aligned} X_{T_{\text{opt}}}^{B,\text{call,down}} &= \max(90-100,0) = 0 \\ X_{T_{\text{opt}}}^{B,\text{call,up}} &= \max(110-100,0) = 10 \\ V_0 &= 0.95\left[(1-0.60)\,(0)+0.60\,(10)\right] \\ &= 5.70 \text{ USD per option} \end{aligned} \tag{5.1.3}$$

The tested domain code does the same two steps: `europeanOptionPayoff` calculates the expiry payoffs, and `backwardInductionValue` discounts them.

5.1.4 Coupon and exercise ordering

In this lesson, a coupon on the expiry date is paid before exercise. So the bond value at expiry is ex-coupon. A contract that is exercised before the coupon, or that transfers the right to the coupon, requires different terminal inputs.

Knowledge check 5.1.1: European bond option

1. A standalone European call on a bond gives its holder which right?
   (a) The right, but not obligation, to buy the named bond at the stated strike and exercise time
   (b) The issuer's right to redeem every outstanding bond
   (c) Automatic ownership of every coupon paid before option exercise
2. An option strike is quoted as a clean bond price and exercise occurs between coupon dates. What is needed before comparing it with a dirty bond value?
   (a) Add the exercise-date accrued interest under the stated convention
   (b) Subtract every coupon paid over the bond's entire life
   (c) No adjustment; clean and dirty prices are identical between coupon dates
3. At expiry a bond has dirty ex-coupon values of USD 78, USD 84, and USD 89 across three nodes. Calculate call payoffs for strike USD 82 and report their sum in USD.
4. One period before expiry, a default-free bond call is worth USD 0 in the down node and USD 6 in the up node. The risk-neutral up weight is 0.4 and the node discount factor is 0.95. Calculate the current option value in USD.

Answers: end of the chapter.

5.1.5 Sources

The bond-option definition and the need to distinguish a cash (dirty) strike from a quoted (clean) strike follow Hull, Chapter 28 §28.1 [1]. This lesson uses an input lattice, not Black's market model, so that the next lesson can model the issuer-default state explicitly.

References

[1] Hull, Options, Futures, and Other Derivatives (8th ed., 2012).

Answers to the knowledge checks

Knowledge check 5.1.1: European bond option

1. **(a)** The right, but not obligation, to buy the named bond at the stated strike and exercise time. This lesson models a standalone holder option, not an embedded issuer call feature.

2. **(a)** Add the exercise-date accrued interest under the stated convention. Payoff inputs must share one price basis. A clean strike becomes a dirty cash strike after adding applicable accrued interest.

3. 9 (accepted within ± 0.000001). The node payoffs are 0, 2, and 7, whose sum is USD 9. The sum is requested only as a compact payoff check, not as a probability-weighted value.

4. 2.28 (accepted within ± 0.000001). Backward induction gives $0.95 \times (0.6 \times 0 + 0.4 \times 6) =$ USD 2.28.

Chapter 5.2

Issuer-default knockout bond options

Interactive version: https://danieltuzes.github.io/credit-and-derivatives/bond-options/issuer-default-knocko

5.2.1 Contract definition

In this lesson, "knockout" has one trigger: if the issuer defaults before the exercise time, the option is extinguished and pays no rebate. This knockout is not a barrier on the market price of the bond.

The underlying bond is a different claim. The bond can still pay recovery after default. So the recovery is part of the bond value, and it is never part of the option value in the default state.

5.2.2 Value the underlying bond first

An alive node is a node at which the issuer has not defaulted. At an alive node with lattice-time-index i and lattice-state-index j, use the local node discount factor $d_{i,j}$, the local risk-neutral up weight $q_{i,j}$, the conditional-node-survival-probability $s_{i,j}$, the next surviving cash flow CF^B_{i+1}, and the local bond recovery $R^B_{i,j}$. The local surviving-bond-continuation-value is the bond value at the next time, conditional on survival. It is the next surviving cash flow plus the expected alive bond value:

$$H^B_{i,j} = CF^B_{i+1} + (1 - q_{i,j})B^{\text{alive}}_{i+1,j} + q_{i,j}B^{\text{alive}}_{i+1,j+1} \tag{5.2.1}$$

The Alive-state bond value is the discounted expectation over survival and default. Survival gives the continuation value from (5.2.1), and default gives the recovery:

$$B^{\text{alive}}_{i,j} = d_{i,j}\left[s_{i,j}H^B_{i,j} + \left(1 - s_{i,j}\right)R^B_{i,j}\right] \tag{5.2.2}$$

The values at each node are ex-cash-flow. The model assumes that recovery is paid at the next lattice date. This timing is a simplification.

5.2.3 Apply the knockout to option continuation

At call expiry, if the issuer has not defaulted, the option value is the call payoff:

$$O^{\text{KO}}_{T,j} = \max\left(B^{\text{alive}}_{T,j} - K, 0\right) \tag{5.2.3}$$

Before expiry, the option value in the default state is zero. So the option value at a node is the discounted, survival-weighted expectation of the option values at the two successor nodes:

$$O^{\mathrm{KO}}_{i,j} = d_{i,j}s_{i,j}\left[\left(1-q_{i,j}\right)O^{\mathrm{KO}}_{i+1,j} + q_{i,j}O^{\mathrm{KO}}_{i+1,j+1}\right] \tag{5.2.4}$$

(5.2.4) has no recovery term, unlike (5.2.2). If the conditional survival probability at the current node is zero, the option value is zero, also when the bond recovery is positive.

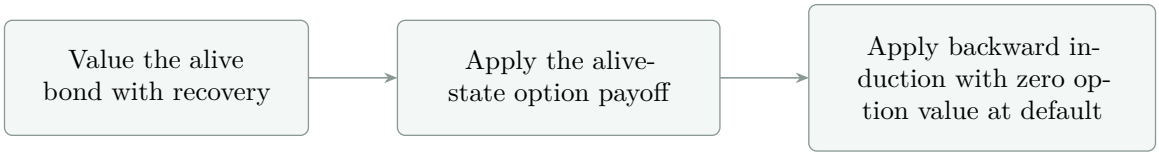


Diagram 5.2.1 **Issuer-default knockout valuation flow.** Value the underlying bond first, including its separate input recovery. At option expiry, apply the payoff only in alive states. Before expiry, discount only the survival-weighted option values, because the option value at default is zero.

5.2.4 Worked example

Consider a lattice with two annual steps. At option expiry, the input values of the final period give alive bond values of USD 82.80 and USD 88.65. With a call strike of USD 85, (5.2.3) gives terminal option values of USD 0 and USD 3.65. At valuation time, the discount factor is 0.95, the conditional survival probability is 0.90, and the up weight is 0.50. The option values are:

$$\begin{aligned} O^{\mathrm{KO}}_{1,0} &= \max(82.80-85,0) = 0 \\ O^{\mathrm{KO}}_{1,1} &= \max(88.65-85,0) = 3.65 \\ O^{\mathrm{KO}}_{0,0} &= 0.95(0.90)\left[(1-0.50)\,(0)+0.50\,(3.65)\right] \\ &= 1.560375 \text{ USD per option} \end{aligned} \tag{5.2.5}$$

The domain function `valueDefaultKnockoutBondOption` performs the same bond recursion and option recursion, in the same order.

5.2.5 Put-call parity with a default knockout

Put-call parity for options without a knockout does not hold unchanged for this contract. In a default state before expiry, the call value minus the put value is zero. So the matching synthetic forward must also be extinguished by the same default event. A default-free forward does not match the options in every state.

5.2.6 Model and source boundary

Hull describes European bond options. Separately, Hull describes CDS forwards and CDS options that cease to exist if the reference entity defaults. This lesson combines the two descriptions for teaching: an option on the issuer's bond that is knocked out by issuer default. The lesson implements the stated contract. It does not claim that all bond options use this convention. The joint lattice for rates and credit is an input; it is not calibrated.

Knowledge check 5.2.1: Issuer-default knockout bond option

1. In this lesson's contract, what happens if the issuer defaults before option exercise?
   (a) The option is extinguished with zero option rebate
   (b) The option holder automatically receives the bond's recovery amount
   (c) The option survives unless the bond price also crosses a barrier
2. The bond pays USD 40 recovery after default, but an otherwise identical call is extinguished on that default. What enters the option's default branch?
   (a) USD 0; the USD 40 belongs only in the bond valuation
   (b) USD 40 because every bond cash flow passes through to the option
   (c) The option strike because exercise is forced
3. At an alive node, the risk-neutral expected next option value conditional on survival is USD 10, conditional survival over the period is 0.90, and the node discount factor is 0.95. Default gives zero option rebate. Calculate the current option value in USD.
4. At an alive node, next option values conditional on survival are USD 2 down and USD 8 up. The risk-neutral up weight is 0.25, conditional survival is 0.80, and the discount factor is 0.96. Calculate the current knockout option value in USD.

Answers: end of the chapter.

5.2.7 Sources

The bond-option cash-price conventions follow Hull, Chapter 28 §28.1. Hull describes the analogous extinguishment at default for CDS forwards and CDS options in Chapter 24 §24.5. Backward induction follows Chapter 12 [1]. A recovery convention must state its base and its timing, as the fixed-income treatment emphasizes [2].

References

[1] Hull, Options, Futures, and Other Derivatives (8th ed., 2012).

[2] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022).

Answers to the knowledge checks

Knowledge check 5.2.1: Issuer-default knockout bond option

1. **(a)** The option is extinguished with zero option rebate. Issuer default is the knockout event. Recovery on the underlying bond is modeled separately and is not an option rebate.
2. **(a)** USD 0; the USD 40 belongs only in the bond valuation. The contract separates underlying recovery from option termination.
3. 8.55 (accepted within ± 0.000001). The value is $0.95 \times [0.90 \times 10 + 0.10 \times 0] =$ USD 8.55.

4. 2.688 (accepted within ± 0.000001). The survival-state expectation is $0.75 \times 2 + 0.25 \times 8 = 3.5$. Multiply by survival and discounting: $0.96 \times 0.80 \times 3.5 = \text{USD } 2.688$.

Appendix A

Notation

The symbols of this book, in the order they first appear.

$X^{B,\text{call}}_{T_{\text{opt}}}$ **Bond call expiry payoff**. Non-negative holder payoff from buying the bond at the dirty strike when economically beneficial. *Units:* stated currency per option at expiry. *In Chapter 5.1 only.*

$B^{\text{dirty}}_{T_{\text{opt}}}$ **Bond option expiry value**. Ex-coupon dirty cash value of the named bond at the European exercise instant. *Units:* stated currency per bond at option expiry. *In Chapter 5.1 only.*

$X^{B,\text{put}}_{T_{\text{opt}}}$ **Bond put expiry payoff**. Non-negative holder payoff from selling the bond at the dirty strike when economically beneficial. *Units:* stated currency per option at expiry. *In Chapter 5.1 only.*

AI **Accrued interest**. Coupon amount attributed to the interval from the previous coupon date through settlement under the stated day-count convention; a positive amount added to the clean price to obtain the dirty invoice price. *Units:* stated currency at settlement.

$V_{i,j}$ **Lattice node value**. Claim value at time row i and state node j obtained by one-period backward induction from its successor nodes, conditional on reaching that node under the input pricing lattice. *Units:* stated currency at the node time.

T_{opt} **Option expiry time**. Future model time when a European option's exercise decision and payoff are determined; a contract date shared by holder and writer, distinct from a bond maturity. *Units:* model-years from the stated valuation time.

K **Option strike price**. Contractual price per unit of underlying used to determine the option's exercise payoff; a positive contractual amount, with payoff signs depending on call or put and holder or writer perspective. *Units:* expiry-time currency per unit of underlying.

$R^{B}_{i,j}$ **Bond default recovery cash**. Non-negative recovery paid on the underlying bond at the next lattice date after default in the period. *Units:* stated currency per bond at the next time. *In Chapter 5.2 only.*

$H^{B}_{i,j}$ **Surviving bond continuation value**. Next-time scheduled bond cash plus risk-neutral expected ex-cash-flow bond value, conditional on period survival. *Units:* stated currency per bond at the next time. *In Chapter 5.2 only.*

CF^{B}_{i+1} **Surviving bond next cash flow**. Coupon or coupon-plus-redemption paid at the next lattice time only if the issuer survives the period. *Units:* stated currency per bond at the next time. *In Chapter 5.2 only.*

$B^{\text{alive}}_{i,j}$ **Alive-state bond value**. Ex-cash-flow value of the defaultable bond at a lattice node conditional on the issuer still being alive, before the subsequent survival and default branches; scheduled cash at the node has already been paid. *Units:* stated currency per bond at the node time.

$s_{i,j}$ **Conditional node survival probability**. Pricing-model probability that the issuer survives the next lattice period, conditional on being alive at the current node; a risk-neutral input, not an unconditional real-world forecast. *Units:* probability between zero and one.

$O^{\text{KO}}_{i,j}$ **Knockout bond option value**. Alive-node value of a European bond option that is extinguished with zero option rebate by issuer default before exercise; a non-negative holder value conditional on the issuer being alive at the node. *Units:* stated currency per option at the node time.

$d_{i,j}$ **Lattice node discount factor**. One-period discount factor at a lattice node: currency at the node per one unit of currency at either successor node one step later. *Units:* current-node currency per next-time currency.

$q_{i,j}$ **Lattice node up weight**. Pricing probability assigned to the up successor, conditional on the current node; in a lattice with issuer default, also conditional on survival to the next lattice time. *Units:* probability between zero and one.

j **Lattice state index**. Integer label for one state node within a time row of a finite recombining lattice; a bookkeeping label under the stated successor ordering, not a probability or state value. *Units:* dimensionless integer index.

i **Lattice time index**. Integer label for one time row in a finite recombining valuation lattice; a bookkeeping label, not a model-year time or currency amount. *Units:* dimensionless integer index.

Appendix B

Abbreviations

CDS Credit default swap, the credit derivative the CDS lessons define and value.

CDX The name of a family of standard credit default swap indices, each a standard portfolio of single-name contracts.

DTS Duration times spread, a spread-risk measure for bonds.

FINRA Financial Industry Regulatory Authority.

IMM International Money Market. In the CDS lessons, IMM dates are the standard maturity dates on the twentieth of March, June, September, and December.

ISDA International Swaps and Derivatives Association.

ISO International Organization for Standardization.

JTD Jump to default, the loss on an immediate default of the reference entity.

UTC Coordinated Universal Time, the time standard the date arithmetic counts calendar days in.