Chapter 1.3

Rate quotes, compounding, and basis points

From *Credit Products Playground*, with changes after version 0.4.0, not yet released. Interactive version: https://danieltuzes.github.io/credit-and-derivatives/foundations/rates-compounding-and-basis-points/

1.3.1 What you will be able to do

After this lesson, you should be able to:

- interpret a nominal annual quote together with its frequency;
- calculate the rate applied in one compounding period;
- convert between decimal rates, percentages, and basis points.

1.3.2 A rate quote needs its compounding convention

The phrase "an annual rate of 6%" does not determine the compounding unless its convention is stated. In this lesson, write the nominal-annual-rate as $j^{(m)}$ and pair it with the compounding-frequency m. Write the resulting periodic-rate as r_m.

In this lesson, t denotes a time-from-valuation: a payment-time without a row index. The valuation-time is 0. Use the local number-of-compounding-periods N for the number of times the periodic rate is applied, and write the resulting accumulation-factor as $A(0,t)$.

Under the nominal-rate convention of this lesson, the periodic rate is the nominal annual rate divided by the compounding frequency:

$$r_m = \frac{j^{(m)}}{m} \tag{1.3.1}$$

This division is a convention of the quote. It is not a conversion rule for every kind of annual rate.

Example 1.3.1: Compounding and basis points

Nominal 6%, compounded semiannually

With $j^{(2)} = 0.06$ and $m = 2$, the periodic rate is:

$$r_2 = \frac{0.06}{2} = 0.03$$

The model applies the periodic rate of 3% once every half-year. Over $t = 1.5$ years, the local number-of-compounding-periods is $N = mt = 3$. So the accumulation-factor is:

$$A(0, 1.5) = (1 + 0.03)^3 = 1.092727$$

One unit at valuation-time accumulates to 1.092727 units at the payment-time 1.5 years later.

Nominal is not effective annual

Under the same convention, the effective annual rate, i.e., the growth over one year of compounding, is:

$$(1.03)^2 - 1 = 0.0609 = 6.09\%$$

So the nominal annual rate of 6% and the effective annual rate of 6.09% are different quantities. The name "annual rate" without a convention does not distinguish them.

Basis-point arithmetic

A basis-point (bp) is a unit for rate differences. One basis point is 0.0001, i.e., 0.01%:

$$25\,\text{bp} = 25 \times 0.0001 = 0.0025 = 0.25\%$$

A change from 4.10% to 4.35% is 0.25 percentage points (25 bp). Write the unit explicitly, because the number 25 without a unit is ambiguous.

1.3.3 Common unit boundary

Table 1.3.1 **One rate in three representations.** The same rate as the decimal the domain code takes, the percentage shown to a learner, and the number of basis points above zero.

Representation	Same rate
Decimal used by domain code	0.0435
Percentage shown to a learner	4.35%
Basis points above zero	435 bp

A lab converts a percentage input to a decimal. The domain functions take decimal rates.

1.3.4 Check your understanding

These direct and transfer items cover quote interpretation, periodic-rate calculation, and basis-point conversion. Each question stays collapsed until you open it; answers and explanations appear once you check.

Knowledge check 1.3.1: Rates, compounding, and basis points

1. Under the lesson convention, what does a 6% nominal annual rate compounded semiannually imply?

(a) A 3% rate in each six-month period

(b) A 6% rate in each six-month period

(c) A 6% rate in each month

2. Two rates are both quoted as 6% nominal annually, but one compounds annually and the other monthly. Which statement is correct under the lesson convention?

   (a) Their per-period rates differ, and their one-year accumulation generally differs

   (b) They have identical per-period rates because both are quoted as 6%

   (c) Both quotes must be 6% effective annual rates

3. Enter the per-period rate as a decimal for a 6% nominal annual rate compounded twice per year.

4. Enter the quarterly rate as a decimal for a 4.8% nominal annual rate compounded four times per year.

5. Express 75 basis points as a decimal rate.

6. A rate rises from 3.25% to 3.70%. How many basis points is the increase? Enter a number of basis points.

Answers: *end of the chapter.*

1.3.5 Model boundary and review note

This lesson does not introduce continuous compounding, a general conversion to effective rates, day counts, or a term structure.

The nominal-quote-plus-frequency convention and the basis-point unit follow Tuckman & Serrat[1]. The lesson stays `draft` pending human confirmation of the printed locators.

References

[1] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). §2.1, rates quoted as annual rates over a term of fixed-length periods, simple or compound; §2.4, eq. 2.17, the periodic-compounding form; a basis point is 0.01%, Ch. 0 footnote and Ch. 3.

Notation

N **Number of compounding periods**. Counts how many times the periodic rate is applied over the stated horizon. Under equal periods, the count equals compounding frequency multiplied by time in years.

$$N = mt$$

$A(0,t)$ **Accumulation factor**. Grows one current unit over a stated future horizon under the selected rate model. *Units:* dimensionless currency-units per current currency-unit.

1 bp **Basis point**. Rate-change unit equal to one hundredth of one percentage point. *Units:* decimal rate.

m **Compounding frequency**. Number of equal compounding periods per year under the stated rate convention, a positive integer fixed by the model convention. *Units:* compounding periods per year.

$j^{(m)}$ **Nominal annual rate**. Annualized rate quote that must be paired with its compounding frequency. *Units:* decimal rate per year.

t_k **Payment time**. Time from the valuation date to one scheduled cash-flow event; a time coordinate, not a calendar date or payment index. *Units:* years from the valuation date.

r_m **Periodic rate**. Rate applied once in each compounding period under the stated convention, derived from the stated nominal annual quote in this model. *Units:* decimal per compounding period.

t **Time from valuation**. A time measured in years from the valuation time; a time coordinate, not a calendar date or a schedule index. With a row index it is a payment time. *Units:* years from the valuation date.

0 **Valuation time**. Common origin from which later model times and present values are measured. *Units:* years from the valuation date.

Answers to the knowledge checks

Knowledge check 1.3.1: Rates, compounding, and basis points

1. **(a)** A 3% rate in each six-month period. For a nominal annual rate $j^{(m)}$ compounded m times per year, the lesson convention uses $r_m = j^{(m)}/m$ in each period: $6\%/2 = 3\%$.

2. **(a)** Their per-period rates differ, and their one-year accumulation generally differs. A nominal quote is incomplete without frequency; the period rate is $r_m = j^{(m)}/m$ and compounding applies it m times.

3. 0.03 (accepted within $\pm 1e-12$). With $j^{(2)} = 0.06$ and $m = 2$, the period rate is $r_2 = 0.06/2 = 0.03$.

4. 0.012 (accepted within $\pm 1e-12$). With $j^{(4)} = 0.048$ and $m = 4$, the quarterly rate is $r_4 = 0.048/4 = 0.012$.

5. 0.0075 (accepted within $\pm 1e-12$). One basis point is 0.0001 in decimal-rate units, so $75 \times 0.0001 = 0.0075$.

6. 45 (accepted within $\pm 1e-9$). The increase is 0.45 percentage point, and each percentage point contains 100 basis points: $0.45 \times 100 = 45$.