Chapter 1.5

Present value of a cash-flow schedule

From *Credit Products Playground*, with changes after version 0.4.0, not yet released. Interactive version: https://danieltuzes.github.io/credit-and-derivatives/foundations/present-value/

1.5.1 What you will be able to do

After this lesson, you should be able to:

- interpret present value from the same perspective as its cash flows;
- discount each dated deterministic amount;
- add discounted amounts without mixing payment dates.

1.5.2 Make cash flows comparable before adding

Use the local present-value-payment-index k to select one row and the local number-of-future-payments n to count the included future rows. At row k, write the signed-cash-flow as CF_k, its payment-time as t_k, and the matching discount-factor as $D(0, t_k)$. Write the schedule's present-value at valuation-time 0 as PV_0.

The present value of the deterministic schedule is the sum of the discounted cash flows:

$$PV_0 = \sum_{k=1}^{n} CF_k D(0, t_k) \tag{1.5.1}$$

The index k and the count n are defined only in this lesson. The other symbols are shared across the course.

Two schedules are compatible when they use the same valuation time, perspective, currency, and discount factors. Let $\mathrm{A} \parallel \mathrm{B}$ be the schedule that contains every row of both schedules A and B. For two compatible schedules, the present value is additive:

$$PV_0(\mathrm{A} \parallel \mathrm{B}) = PV_0(\mathrm{A}) + PV_0(\mathrm{B}) \tag{1.5.2}$$

Example 1.5.1: Present values of dated cash flows

One payment

A deterministic receipt of USD 1,000 is paid after two years, and $D(0,2) = 0.9070294785$. The present value of the receipt is:

$$PV_0 = 1000 \times 0.9070294785 = \text{USD } 907.0294785 \approx \text{USD } 907.03$$

The present value is in USD at valuation-time, from the holder's perspective.

Mixed signed cash flows

Suppose the holder receives USD 100 at t_1, pays USD 40 at t_2, and receives USD 300 at t_3. The discount factors are 0.97, 0.93, and 0.88.

Payment index k	Cash flow CF_k, USD	Discount factor $D(0,t_k)$	Discounted cash flow $CF_k D(0,t_k)$, USD
1	+100	0.97	+97.00
2	-40	0.93	-37.20
3	+300	0.88	+264.00

The present value is the sum of the discounted cash flows:

$$PV_0 = 97.00 - 37.20 + 264.00 = \text{USD } 323.80$$

Discounting does not change the sign of a cash flow. Reversing the perspective would change the sign of the present value.

Additivity

Combining two schedules keeps every row, including duplicate payments. For compatible schedules, a property test of the domain code checks (1.5.2). The compatibility conditions are necessary: present values in different currencies, or at different valuation dates, cannot be added without another model.

Figure 1.5.1 shows the one-payment case. The discount-factor for one time-from-valuation t is calculated from one discounting-lab-rate with annual compounding. One discounting-lab-cash-flow multiplied by that discount factor gives the present-value at valuation-time 0.

The tested domain code calculates every value in the figure. The figure states the rate as a percentage. The calculation converts it to a decimal first.

1.5.3 Check your understanding

These items separate interpretation from calculation and include an unfamiliar multi-payment case. Each question stays collapsed until you open it; answers and explanations appear once you check.

Knowledge check 1.5.1: Present value

1. Under stated discount factors and a valuation time of 0, what does the present value of a future cash-flow stream represent?

   (a) The valuation-time sum of the individually discounted cash flows

   (b) The undiscounted total of every future cash flow

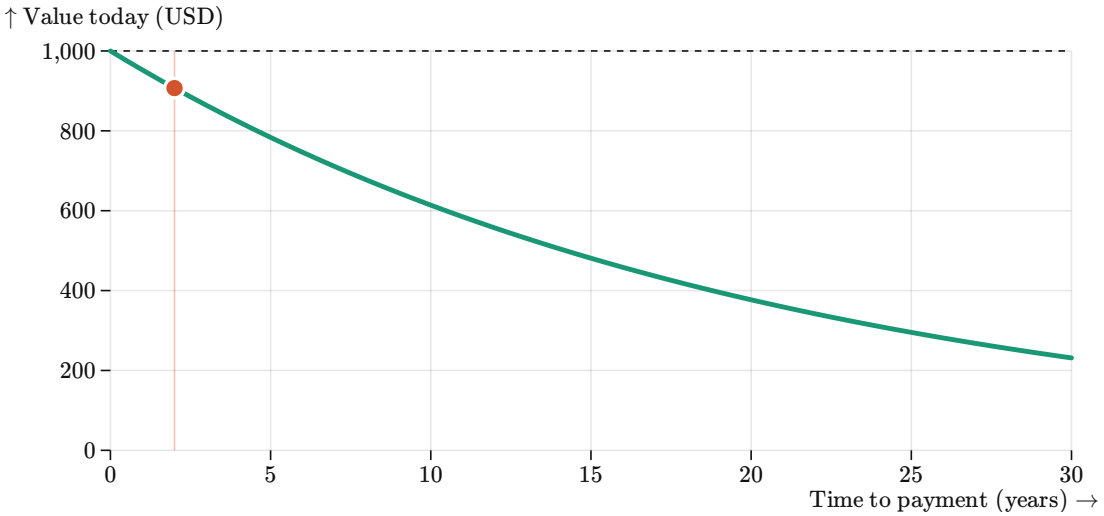


Figure 1.5.1 **One-payment discounting explorer.** The value today, in USD, of one future payment for every time to payment from 0 to 30 years, at one annual rate with annual compounding. The dot and the vertical line mark the time to payment shown; the dashed line marks the undiscounted payment. Shown for a future payment of USD 1,000 in 2 years at an annual rate of 5%: value today USD 907.03.

(c) Only the cash flow at the latest payment time

2. Every cash flow in a stream is positive. A second stated discount-factor set is smaller at every payment time. What happens to present value?

   (a) Present value is lower

   (b) Present value is higher

   (c) Present value is unchanged

3. From a holder's perspective, a deterministic USD 1,000 receipt at t = 2 years has discount factor D(0,2) = 0.9070294785. What is its time-zero present value in USD?

4. From a holder's perspective, two deterministic USD 100 receipts at different future dates have matching discount factors 0.97 and 0.93. What is their total time-zero present value in USD?

Answers: *end of the chapter.*

1.5.4 Model boundary and review note

This lesson covers deterministic present value only. It omits curves, calendars, uncertainty, credit, recovery, liquidity, tax, and funding.

The rule (1.5.1) follows Tuckman & Serrat[1]; Hull states the same principle[2]. The lesson stays `draft` pending human confirmation of the printed locators.

References

[1] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). §1.2, price as the sum of each cash flow times its discount factor (eqs. 1.1-1.3); full price equals present value (eq. 1.5).

[2] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 4, "Bond Pricing".

Notation

C **Discounting lab cash flow**. The one dated amount the discounting lab discounts. It is page-local to the lab's one-payment model; the lesson's schedule notation writes an indexed amount instead. *Units:* stated currency at payment time.

$$PV_0 = C \cdot D(0,t)$$

y **Discounting lab rate**. The one annual rate the discounting lab compounds to build each discount factor. It is page-local to the lab's toy model; elsewhere this lesson takes every discount factor as given rather than deriving it from a rate. *Units:* decimal rate per year.

$$D(0,t) = (1+y)^{-t}$$

n **Number of future payments**. The final index in the finite present-value sum. It counts included future rows and does not itself specify maturity in years.

$$n \in \mathbb{N}$$

k **Present value payment index**. Selects one dated cash flow in the finite schedule. This lesson-local index keeps sequence bookkeeping separate from the associated time measured in years.

$$k \in \{1, \dots, n\}$$

$A(0,t)$ **Accumulation factor**. Grows one current unit over a stated future horizon under the selected rate model. *Units:* dimensionless currency-units per current currency-unit.

m **Compounding frequency**. Number of equal compounding periods per year under the stated rate convention, a positive integer fixed by the model convention. *Units:* compounding periods per year.

$D(0,t)$ **Discount factor**. Converts one deterministic future unit into value at valuation time. *Units:* current currency-units per future currency-unit.

$j^{(m)}$ **Nominal annual rate**. Annualized rate quote that must be paired with its compounding frequency. *Units:* decimal rate per year.

t_k **Payment time**. Time from the valuation date to one scheduled cash-flow event; a time coordinate, not a calendar date or payment index. *Units:* years from the valuation date.

r_m **Periodic rate**. Rate applied once in each compounding period under the stated convention, derived from the stated nominal annual quote in this model. *Units:* decimal per compounding period.

PV_0 **Present value**. Combines dated signed cash flows into one value at valuation time, using the same holder perspective as the signed cash flows. *Units:* stated currency at the valuation time.

CF_k **Signed cash flow**. Amount received or paid at one event from the stated holder perspective; positive means received and negative means paid by that holder. *Units:* stated currency units.

t **Time from valuation**. A time measured in years from the valuation time; a time coordinate, not a calendar date or a schedule index. With a row index it is a payment time. *Units:* years from the valuation date.

0 **Valuation time**. Common origin from which later model times and present values are measured. *Units:* years from the valuation date.

Answers to the knowledge checks

Knowledge check 1.5.1: Present value

1. **(a)** The valuation-time sum of the individually discounted cash flows. Present value puts each dated cash flow at the common valuation time using its discount factor, then adds the contributions.

2. **(a)** Present value is lower. Each positive cash flow is multiplied by a smaller non-negative factor, so every contribution and their sum are lower.

3. 907.0294785 (accepted within ± 0.01). Multiply the future cash flow by its discount factor.

4. 190 (accepted within ± 0.01). Discount each dated payment and then add: $100 \times 0.97 + 100 \times 0.93 = 190$.