Chapter 1.4

Discount factors

From *Credit Products Playground*, with changes after version 0.4.0, not yet released. Interactive version: https://danieltuzes.github.io/credit-and-derivatives/foundations/discount-factors/

1.4.1 What you will be able to do

After this lesson, you should be able to:

- explain the units and meaning of a discount factor;
- calculate one from a stated nominal rate, frequency, and time;
- distinguish discounting from compounding and from payment probability.

1.4.2 Undo accumulation

In this lesson, t denotes a time-from-valuation: a payment-time without a row index. It is measured in years from valuation-time 0. Write the compounding-frequency as m, the periodic-rate as r_m, the accumulation-factor as $A(0,t)$, and the discount-factor as $D(0,t)$.

The accumulation factor $A(0,t)$ is the amount at time t from one unit invested at time zero. The discount factor is its reciprocal:

$$D(0,t) = \frac{1}{A(0,t)} = \left(1 + r_m\right)^{-mt} \tag{1.4.1}$$

The discount factor, the periodic rate, and the compounding frequency keep their different meanings, also when they appear in one expression.

1.4.3 Meaning before arithmetic

Write the local discounted-unit-value as $V_0(1_t)$: the value at valuation time of one deterministic unit paid at t. If $D(0,2) = 0.94$, the value at valuation time of one unit paid at $t = 2$ years is:

$$V_0(1_2) = 1 \times D(0,2) = 0.94 \tag{1.4.2}$$

The number 0.94 is not an annual interest rate of 94%. It is also not a probability of 94% that the payment occurs.

Example 1.4.1: Discount factors from rates and curves

Annual compounding

With a nominal-annual-rate of 5%, $m = 1$, and $t = 2$, the discount factor is:

$$D(0,2) = \left(1 + \frac{0.05}{1}\right)^{-1\times 2} = \frac{1}{1.05^2} \approx 0.907029$$

Semiannual compounding

With a nominal annual rate of 6%, compounded twice per year, the discount factor for 1.5 years is:

$$D(0,1.5) = \left(1 + \frac{0.06}{2}\right)^{-2(1.5)} = \frac{1}{1.03^3} \approx 0.915142$$

The horizon of 1.5 years contains three compounding periods. The exponent 1.5 would be wrong, because it counts years, not half-year periods.

Read a small curve

With the same nominal annual rate of 6%, compounded semiannually, the discount factors are:

Time t, years	Periods mt	Discount factor $D(0,t)$
0.0	0	1.000000
0.5	1	0.970874
1.0	2	0.942596
1.5	3	0.915142

If the rate is non-negative, the discount factor does not increase with the payment time. If the rate is positive, the discount factor decreases with the payment time.

1.4.4 Check your understanding

These items test meaning, annual calculation, and a semiannual transfer case. Each question stays collapsed until you open it; answers and explanations appear once you check.

Knowledge check 1.4.1: Discount factors

1. If D(0,2) = 0.94, what does 0.94 mean?
   (a) The value today of one unit paid in two years
   (b) A 94% annual interest rate
   (c) The probability that payment occurs
2. For the same positive future cash flow, which discount factor gives the lower present value?
   (a) 0.91
   (b) 0.97
   (c) They give the same value

3. Calculate the dimensionless two-year discount factor from a 5% nominal annual rate compounded annually.

4. Calculate the dimensionless 1.5-year discount factor from a 6% nominal annual rate compounded semiannually.

Answers: *end of the chapter.*

1.4.5 Model boundary and review note

The rate is the same for all times, and the cash flows are deterministic. This lesson does not teach spot curves, interpolation, day counts, default, recovery, liquidity, or funding.

The discount-factor definition (1.4.1) follows Tuckman & Serrat[1]; Hull gives the same interpretation, the present value of one unit paid in the future[2]. The lesson stays `draft` pending human confirmation of the printed locators.

References

[1] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). §1.2, the discount factor as the value today of one unit paid at t; §2.4, eqs. 2.17-2.19, the semiannually compounded form.

[2] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 4, "Bond Pricing".

Notation

$V_0(1_t)$ **Discounted unit value**. A local reading aid that makes discount-factor units explicit. It equals one future currency unit multiplied by the discount factor for its payment time. *Units:* valuation currency-units per future currency-unit.

$$V_0(1_t) = 1_t D(0,t)$$

$A(0,t)$ **Accumulation factor**. Grows one current unit over a stated future horizon under the selected rate model. *Units:* dimensionless currency-units per current currency-unit.

m **Compounding frequency**. Number of equal compounding periods per year under the stated rate convention, a positive integer fixed by the model convention. *Units:* compounding periods per year.

$D(0,t)$ **Discount factor**. Converts one deterministic future unit into value at valuation time. *Units:* current currency-units per future currency-unit.

$j^{(m)}$ **Nominal annual rate**. Annualized rate quote that must be paired with its compounding frequency. *Units:* decimal rate per year.

t_k **Payment time**. Time from the valuation date to one scheduled cash-flow event; a time coordinate, not a calendar date or payment index. *Units:* years from the valuation date.

r_m **Periodic rate**. Rate applied once in each compounding period under the stated convention, derived from the stated nominal annual quote in this model. *Units:* decimal per compounding period.

t **Time from valuation**. A time measured in years from the valuation time; a time coordinate, not a calendar date or a schedule index. With a row index it is a payment time. *Units:* years from the valuation date.

0 **Valuation time**. Common origin from which later model times and present values are measured. *Units:* years from the valuation date.

Answers to the knowledge checks

Knowledge check 1.4.1: Discount factors

1. **(a)** The value today of one unit paid in two years. A discount factor converts a future unit into today's value; it is neither an interest rate nor a survival probability.

2. **(a)** 0.91. Present value is the cash flow multiplied by the discount factor, so the smaller positive factor produces the smaller value.

3. 0.9070294784580498 (accepted within ± 0.000001). $D(0,2) = \frac{1}{1.05^2}$.

4. 0.9151416593531596 (accepted within ± 0.000001). The periodic rate is $0.06/2 = 0.03$ and 1.5 years contains three half-year periods, so $D(0,1.5) = \frac{1}{1.03^3}$.