Chapter 4.1

Forward contracts, delivery price, and value

From *Credit Products Playground*, with changes after version 0.4.0, not yet released. Interactive version: https://danieltuzes.github.io/credit-and-derivatives/derivatives/forward-contracts-and-value/

4.1.1 What you will be able to do

After this lesson, you should be able to:

- read the long and short obligations of a forward;
- distinguish delivery price, current forward price, and contract value;
- calculate a fair delivery price with deterministic income;
- calculate the signed value of an existing forward.

4.1.2 An obligation, not an option

The exchange occurs at the forward-delivery-time T_{fwd}. Under the physical-settlement convention, the long pays the forward-delivery-price K_{fwd} and receives one unit of the asset. If the derivative-underlying-value at delivery is $S_{T_{\text{fwd}}}$, the local long-forward-expiry-payoff per unit is the asset value minus the delivery price:

$$\Pi^{\text{long}}_{T_{\text{fwd}}} = S_{T_{\text{fwd}}} - K_{\text{fwd}} \tag{4.1.1}$$

The payoff of the short is the negative of the payoff of the long. Hull distinguishes this bilateral obligation from an option holder's exercise right and gives the corresponding long and short payoff directions[1].

4.1.3 Fair delivery price at inception

The forward-price $F_{0,T_{\text{fwd}}}$ is the delivery price that would make a new contract worth zero at valuation-time 0. Suppose the underlying is worth S_0 and the underlying-income-present-value paid

before delivery is I_0. The long does not own the asset before delivery and does not receive the income. So the prepaid value of the asset delivered is $S_0 - I_0$.

Under the stated cash-and-carry assumptions, with the delivery-time discount-factor $D(0, T_{\text{fwd}})$, the law of one price gives the forward price:

$$F_{0,T_{\text{fwd}}} = \frac{S_0 - I_0}{D(0, T_{\text{fwd}})} \tag{4.1.2}$$

Hull derives the no-income and known-income forward prices by matching a spot purchase and financing strategy to the forward exchange [2].

For $S_0 = 100$, $I_0 = 4$, and $D(0, T_{\text{fwd}}) = 0.95$, the forward price is:

$$F_{0,T_{\text{fwd}}} = \frac{100 - 4}{0.95} = 101.0526316 \tag{4.1.3}$$

The delivery price is not a premium paid at inception. A new contract with a fair delivery price has signed value zero, because its delivery price equals the current forward price.

4.1.4 An existing forward can gain or lose value

After inception, K_{fwd} stays fixed, but the current forward price can change. For the local forward-contract-quantity N, the forward-contract-value to the long at time zero is the discounted difference between the current forward price and the delivery price:

$$V_0^{\text{fwd}} = ND(0, T_{\text{fwd}}) \left(F_{0,T_{\text{fwd}}} - K_{\text{fwd}} \right) \tag{4.1.4}$$

If $N = 10$, the current forward price is 98, the contract delivery price is 102, and the discount factor is 0.94, the value to the long is:

$$V_0^{\text{fwd}} = 10(0.94)(98 - 102) = -37.60 \tag{4.1.5}$$

The short's value is USD +37.60 under the same inputs. Hull derives this discounted forward-price difference for an existing contract [3].

Knowledge check 4.1.1: Forward contracts and value

1. What does the long side of a physically settled forward contract agree to do at delivery?
   (a) Buy the underlying for the fixed delivery price
   (b) Choose whether to buy, with no obligation
   (c) Receive the delivery price as an inception premium
2. A long forward has delivery price USD 80 and the underlying is worth USD 74 at delivery. What is the long's payoff per unit?
   (a) USD -6
   (b) USD +6
   (c) USD 0 because the long can decline delivery
3. An investment asset is worth USD 100, pays no income before delivery, and D(0,T) = 0.95. What fair delivery price gives a new forward zero value, in USD per unit?

4. A bond's current full price is USD 900. A known coupon before delivery has present value USD 40, and D(0,T) = 0.96. What is the fair full forward delivery price in USD?

5. The current forward price is USD 110, an existing long forward's delivery price is USD 105, and D(0,T) = 0.97. What is the contract value to the long per unit, in USD?

6. A long forward covers 10 units. The current forward price is USD 98 per unit, its fixed delivery price is USD 102, and D(0,T) = 0.94. What is its signed value to the long in USD?

Answers: *end of the chapter.*

4.1.5 Model boundary and review note

This lesson uses a toy cash-and-carry model for an investment asset with known income and deterministic discounting. It does not model futures-style daily settlement, repo specialness, collateral, counterparty credit, taxes, borrowing asymmetry, delivery options, or an asset that cannot be financed or shorted on the assumed terms.

The formulas are implemented in a pure tested domain module. Sources, notation, examples, code, and answer keys remain `draft` pending independent human review.

References

[1] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 1 §1.3, printed pp. 5-7, forward contracts and payoffs.

[2] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 5 §§5.3-5.5, printed pp. 103-108, cash-and-carry forward pricing and known income.

[3] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 5 §5.7, printed pp. 109-111, valuation of existing forward contracts.

Notation

N **Forward contract quantity**. Positive number of underlying units delivered under the forward contract. Contract value and payoff scale linearly with this quantity. *Units:* units of underlying.

$$N > 0$$

$\Pi^{\text{long}}_{T_{\text{fwd}}}$ **Long forward expiry payoff**. Signed delivery-time value received by the long after exchanging the fixed delivery payment for the underlying. Per unit it equals underlying value at delivery minus the contractual delivery price. *Units:* stated currency at forward delivery time.

$A(0,t)$ **Accumulation factor**. Grows one current unit over a stated future horizon under the selected rate model. *Units:* dimensionless currency-units per current currency-unit.

m **Compounding frequency**. Number of equal compounding periods per year under the stated rate convention, a positive integer fixed by the model convention. *Units:* compounding periods per year.

S_t **Derivative underlying value**. Value at model time t of one unit of the asset or claim named as the derivative's underlying; a positive quoted value, while a position in the underlying carries its own signed quantity. *Units:* stated currency per unit of underlying at model time.

$D(0,t)$ **Discount factor**. Converts one deterministic future unit into value at valuation time. *Units:* current currency-units per future currency-unit.

V_t^{fwd} **Forward contract value**. Signed current value of an existing forward from the named counterparty's perspective; positive to the long when the current forward price exceeds the contract's fixed delivery price under the lesson model. *Units:* stated currency at model time.

K_{fwd} **Forward delivery price**. Contractual currency amount per unit of underlying paid by the long at delivery and received by the short. *Units:* delivery-time currency per unit of underlying.

T_{fwd} **Forward delivery time**. Future model time when the forward counterparties exchange the underlying and delivery payment; a contract date shared by the long and short, not the underlying asset's maturity. *Units:* model-years from the stated valuation time.

$F_{t,T_{\text{fwd}}}$ **Forward price**. Delivery price that would give a newly struck forward for the stated delivery time zero current value; a quoted contract rate rather than a cash amount received at quotation time. *Units:* delivery-time currency per unit of underlying.

$j^{(m)}$ **Nominal annual rate**. Annualized rate quote that must be paired with its compounding frequency. *Units:* decimal rate per year.

r_m **Periodic rate**. Rate applied once in each compounding period under the stated convention, derived from the stated nominal annual quote in this model. *Units:* decimal per compounding period.

I_0 **Underlying income present value**. Valuation-time value of deterministic cash income paid by the underlying after valuation and before the contract horizon, forward delivery or the matched option expiry. It is subtracted from spot value because the forward or option position does not receive that income. *Units:* stated currency at valuation time per unit of underlying.

0 **Valuation time**. Common origin from which later model times and present values are measured. *Units:* years from the valuation date.

Answers to the knowledge checks

Knowledge check 4.1.1: Forward contracts and value

1. **(a)** Buy the underlying for the fixed delivery price. A forward is a bilateral obligation. At delivery the long pays the fixed delivery price and receives the underlying under the stated physical-settlement convention.

2. **(a)** USD -6. The long payoff is underlying value minus delivery price: 74 - 80 = USD -6. Unlike an option holder, the forward long cannot simply decline delivery in this model.

3. 105.2631578947 (accepted within ± 0.000001). The prepaid forward is USD 100. Divide by the delivery-date discount factor: 100 / 0.95 = USD 105.2631578947.

4. 895.8333333333 (accepted within ± 0.000001). Subtract income received before delivery, then carry the prepaid value to delivery: (900 - 40) / 0.96 = USD 895.8333333333.

5. 4.85 (accepted within ± 0.000001). Discount the delivery-date advantage: $0.97 \times (110 - 105)$ = USD 4.85.

6. -37.6 (accepted within ± 0.000001). The signed value is $10 \times 0.94 \times (98 - 102)$ = USD -37.60.