# Chapter 4.2

# European option contracts and payoffs

From *Credit Products Playground*, with changes after version 0.4.0, not yet released. Interactive version: https://danieltuzes.github.io/credit-and-derivatives/derivatives/european-option-contracts-and-payoffs/

## 4.2.1 What you will be able to do

After this lesson, you should be able to:

- identify the holder, writer, underlying, strike, and expiry of a European option;
- calculate long and short call and put payoffs;
- distinguish current value, premium, expiry payoff, and profit.

## 4.2.2 Rights and obligations

A European call gives its holder the right, but not the obligation, to buy one unit of the underlying for the option-strike-price $K$ at the option-expiry-time $T_{\text{opt}}$. A European put gives the holder the corresponding right to sell. If the holder exercises, the writer must carry out the other side of the exchange. Hull introduces call and put rights and European exercise [1].

## 4.2.3 Expiry payoff

Let the derivative-underlying-value at expiry be $S_{T_{\text{opt}}}$. The local long-call-expiry-payoff and long-put-expiry-payoff, per unit, are

$$\Pi^{\text{call}}_{T_{\text{opt}}} = \max\Big(S_{T_{\text{opt}}} - K, 0\Big) \tag{4.2.1}$$

$$\Pi^{\text{put}}_{T_{\text{opt}}} = \max\Big(K - S_{T_{\text{opt}}}, 0\Big) \tag{4.2.2}$$

The maximum with zero expresses the holder's right not to exercise when exercise would give a negative payoff. A writer's signed payoff is the negative of the matching holder payoff.

For $S_{T_{\text{opt}}} = 115$ and $K = 100$, one call pays 15 and one put pays zero. For $S_{T_{\text{opt}}} = 70$ and $K = 85$, one put pays 15. A writer of three such puts has a signed payoff of $-45$.

### 4.2.4 Value, premium, payoff, and profit are different

The European call value $c_t$ or European put value $p_t$ is a current value. At inception, the holder pays the local option-premium-paid $\pi_0$, under the stated settlement convention. The payoff is determined later, by the state at expiry.

If financing is ignored, the local option-holder-expiry-profit of a call is the payoff minus the premium:

$$G_{T_{\text{opt}}} = \Pi^{\text{call}}_{T_{\text{opt}}} - \pi_0 \tag{4.2.3}$$

For example, a call bought for 6 that later pays 9 has a payoff of 9 and a profit of 3, if financing is ignored. A profit calculation that includes financing first moves the premium and all other cash flows to one common date. Hull's option examples distinguish premium, exercise, payoff, and profit [2].

**Knowledge check 4.2.1: European option contracts and payoffs**

1. What right does the holder of a European call have in this lesson?
   (a) The right to buy the underlying at the strike at expiry
   (b) The right to sell the underlying at any time before expiry
   (c) An obligation to buy the underlying at expiry
2. Who has the exercise right and who must perform if a European put is exercised?
   (a) The holder chooses; the writer must perform
   (b) The writer chooses; the holder must perform
   (c) Both sides independently choose whether to perform
3. Two European calls each have strike USD 100. The underlying is worth USD 115 at expiry. What is the total holder payoff in USD?
4. A trader writes three European puts with strike USD 85. The underlying is worth USD 70 at expiry. What is the writer's signed payoff in USD?
5. A call is purchased today for a USD 6 premium and later pays USD 9 at expiry. Ignoring financing, which quantities are correctly labeled?
   (a) Expiry payoff USD 9; profit USD 3
   (b) Expiry payoff USD 3; profit USD 9
   (c) Both payoff and profit are USD 9
6. An out-of-the-money option expires with zero payoff. Does that imply its value was zero on every earlier date?
   (a) No, earlier value reflected the remaining possible future payoffs
   (b) Yes, a zero realized payoff makes every earlier value zero
   (c) Yes, option value always equals expiry intrinsic value

*Answers:* *end of the chapter.*

### 4.2.5 Model boundary and review note

This lesson covers European calls and puts only. It does not value them or cover American or Bermudan exercise, automatic exercise, assignment, settlement delay, corporate actions, collateral, counterparty default, or barrier and other path-dependent terms.

The payoff functions are implemented in a pure tested domain module. Sources, notation, examples, code, and answer keys remain `draft` pending independent human review.

## References

[1] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 1 §1.5, printed pp. 7-9, call and put option rights and European exercise.

[2] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 9 practice questions 9.1-9.2 and 9.9-9.10, printed pp. 211-212, payoff versus profit.

## Notation

$\Pi^{\text{call}}_{T_{\text{opt}}}$ **Long call expiry payoff**. Non-negative amount received by the holder of a European call at expiry. Per unit it is the positive part of underlying expiry value minus strike. *Units:* stated currency at option expiry.

$\Pi^{\text{put}}_{T_{\text{opt}}}$ **Long put expiry payoff**. Non-negative amount received by the holder of a European put at expiry. Per unit it is the positive part of strike minus underlying expiry value. *Units:* stated currency at option expiry.

$G_{T_{\text{opt}}}$ **Option holder expiry profit**. Expiry payoff less the premium carried to expiry under the explicitly stated financing convention. In the simplest examples financing is ignored, so the initially paid premium is subtracted directly. *Units:* stated currency at option expiry.

$\pi_0$ **Option premium paid**. Positive valuation-time amount paid by the option holder to acquire the contractual right. Its financing convention must be stated before converting an expiry payoff into profit. *Units:* stated currency at premium payment time.

$c_t$ **European call value**. Current non-negative value to the holder of a European call under the stated model, before any financing or transaction costs. *Units:* stated currency at model time.

$S_t$ **Derivative underlying value**. Value at model time t of one unit of the asset or claim named as the derivative's underlying; a positive quoted value, while a position in the underlying carries its own signed quantity. *Units:* stated currency per unit of underlying at model time.

$T_{\text{opt}}$ **Option expiry time**. Future model time when a European option's exercise decision and payoff are determined; a contract date shared by holder and writer, distinct from a bond maturity. *Units:* model-years from the stated valuation time.

$K$ **Option strike price**. Contractual price per unit of underlying used to determine the option's exercise payoff; a positive contractual amount, with payoff signs depending on call or put and holder or writer perspective. *Units:* expiry-time currency per unit of underlying.

$p_t$ **European put value**. Current non-negative value to the holder of a European put under the stated model, before any financing or transaction costs. *Units:* stated currency at model time.

## Answers to the knowledge checks

**Knowledge check 4.2.1: European option contracts and payoffs**

1. **(a)** The right to buy the underlying at the strike at expiry. A European call holder has a right, not an obligation, to buy at the strike at the single expiry time.

2. **(a)** The holder chooses; the writer must perform. The option premium buys the holder the exercise right; the writer bears the corresponding obligation under the stated contract.

3. 30 (accepted within $\pm 0.000001$). Each call pays max(115 - 100, 0) = USD 15. Two calls pay USD 30.

4. $-45$ (accepted within $\pm 0.000001$). Each holder receives max(85 - 70, 0) = USD 15, so the writer pays $3 \times 15$ = USD 45 and has payoff USD -45.

5. **(a)** Expiry payoff USD 9; profit USD 3. The contract payoff is USD 9. Ignoring financing, profit subtracts the USD 6 premium and is USD 3.

6. **(a)** No, earlier value reflected the remaining possible future payoffs. Before expiry, more than one future state may remain possible. A realized zero payoff does not retrospectively erase the value of the contingent right.