Chapter 6.2

Recovery and one-period risky present value

From *Credit Products Playground*, with changes after version 0.4.0, not yet released. Interactive version: https://danieltuzes.github.io/credit-and-derivatives/credit/recovery-and-risky-present-value/

6.2.1 What you will be able to do

After this lesson, you should be able to:

- interpret recovery as a fraction of a named base paid at a named time;
- calculate loss given default as the complement of recovery;
- calculate present value in one explicitly labeled recovery-of-par-paid-at-maturity model.

6.2.2 Name the recovery convention before calculating

This lesson uses one recovery convention. A claim has a positive local recovery-par-amount F and one local recovery-payment-time T, measured from valuation-time 0.

- If the reference entity survives through T, the holder receives F at T.
- If the reference entity defaults at or before T, the holder receives RF at T, where the recovery-rate R is deterministic.

The survival payment and the default payment are both paid at T. This timing is a model assumption, not a description of an actual instrument. This lesson does not teach recovery paid at default, recovery of market value, settlement delay, or a CDS settlement mechanism.

Tuckman and Serrat define a bond recovery rate relative to the face amount. They also identify the complementary loss fraction in their credit-loss discussion [1]. Payment of the recovery at maturity is a convention of this lesson. The lesson does not claim that it is the only recovery convention.

6.2.3 Recovery and loss given default are complements here

The loss-given-default fraction is defined as:

$$\text{LGD} = 1 - R, \qquad 0 \leq R \leq 1 \tag{6.2.1}$$

For example, $R = 0.40$ gives $\text{LGD} = 0.60$ in this toy model. The value 0.40 is chosen only for the arithmetic. The lesson makes no claim that 40% is a market convention, a forecast, or an appropriate assumption for any real entity.

6.2.4 Calculate the expected maturity amount, then discount it

The input survival-probability is $S^{\mathbb{Q}}(0,T)$ under the risk-neutral-probability-measure $\mathbb{Q}$. In the one-default model, the probability of default by T is $1 - S^{\mathbb{Q}}(0,T)$.

For the single interval from 0 through T, $1 - S(0,T)$ is the interval-default-probability. The expected amount paid at T under $\mathbb{Q}$ is the survival payment and the default payment, each multiplied by its probability:

$$F\left[S\left(0,T\right) + R\left(1 - S\left(0,T\right)\right)\right] \tag{6.2.2}$$

This is the two-event case of the expectation from disjoint events in the probability lesson: survival pays F, and default pays RF. Shreve gives the expectation of a finite-valued random variable[2]. Tuckman and Serrat weight credit cash flows by survival and default probabilities and then discount them[3].

The survival payment and the default payment are both paid at T. So one input discount-factor $D(0,T)$ applies to both. The recovery-of-par-present-value is the discounted expected amount:

$$PV_0^{\text{RoP}} = F\,D(0,T)\left[S^{\mathbb{Q}}\left(0,T\right) + R\left(1 - S^{\mathbb{Q}}\left(0,T\right)\right)\right] \tag{6.2.3}$$

This present-value is calculated from the perspective of the holder of the claim: the modeled future receipts and their value are positive. A purchase price paid by the holder would have the opposite cash-flow sign.

Example 6.2.1: Recovery-of-par present values

Partial recovery

Let $F = \text{USD } 1{,}000$, $D(0,T) = 0.96$, $S(0,T) = 0.92$, and $R = 0.40$. The expected fraction of par paid at maturity is:

$$0.92 + 0.40(1 - 0.92) = 0.952$$

Multiplying this fraction by the par amount and the discount factor gives the present value:

$$PV_0^{\text{RoP}} = 1000(0.96)(0.952) = \text{USD } 913.92$$

The expected amount at maturity, before discounting, is USD 952.00. The discount factor converts this amount to its value at valuation time.

Zero-recovery boundary

Let $F = \text{USD } 1{,}000$, $D(0,T) = 0.94$, $S(0,T) = 0.90$, and $R = 0$. The default payment is zero, so the present value is:

$$PV_0^{\text{RoP}} = 1000(0.94)\left[0.90 + 0\,(0.10)\right] = \text{USD } 846.00$$

With zero recovery, the present value is the discounted expected survival payment. This example makes no statement about real recovery outcomes.

Full-recovery boundary

Keep $F = \text{USD } 1{,}000$, $D(0,T) = 0.94$, and $S(0,T) = 0.90$, but set $R = 1$. The survival payment and the default payment are now both the full par amount, paid at T:

$$PV_0^{\text{RoP}} = 1000(0.94)\left[0.90 + 1\,(0.10)\right] = \text{USD } 940.00$$

With full recovery, the present value does not depend on the survival probability. The reason is that both modeled events pay the same amount on the same date. This result holds only under the timing convention of this lesson.

6.2.5 Check your understanding

The assessment tests the meaning of recovery, loss-given-default arithmetic, and direct and transfer calculations under the same one-period timing convention.

Knowledge check 6.2.1: Recovery and risky present value

1. In the lesson's one-period recovery-of-par model, what does a recovery rate of 40% mean if default occurs by maturity?
   (a) A recovery cash flow equal to 40% of par is paid at maturity
   (b) A recovery cash flow equal to 40% of the current market price is paid at maturity
   (c) A recovery cash flow equal to 60% of par is paid at maturity
2. A one-period claim has USD 2,000 par, a current price of USD 1,850, and a 25% recovery-of-par rate. Under the lesson model, which positive recovery cash flow is used after default?
   (a) USD 500 at maturity
   (b) USD 462.50 at maturity
   (c) USD 1,500 at maturity
3. Enter loss given default as a decimal when the recovery rate is 35% (0.35).
4. A recovery-of-par model pays USD 900 after default on a claim with USD 1,500 par. Enter loss given default as a decimal.
5. A one-period defaultable zero-coupon claim has USD 1,000 par, survival probability 0.90 to maturity, recovery-of-par rate 0.40 after default, and maturity discount factor 0.95. Both the survival payoff and any recovery are paid at maturity. What is its time-zero present value in USD?
6. A one-period defaultable zero-coupon claim has USD 2,500 par, cumulative default probability 0.08, loss given default 0.55, and maturity discount factor 0.92. The recovery-of-par amount after default is paid at maturity. What is its time-zero present value in USD?

Answers: *end of the chapter.*

6.2.6 Model boundary and review note

The model has one maturity date, one deterministic recovery rate, one input survival probability, and one input discount factor. Recovery is a fraction of par paid at maturity, also when default occurs earlier. The lesson omits:

- settlement at the default time;
- coupons and multiple intervals;
- stochastic recovery and recovery of market value;
- dependence between interest rates and default;
- calibration;
- liquidity, funding, taxes, and counterparty risk.

All materially changed content and source locators remain ‘draft‘ pending independent human review.

References

[1] Tuckman & Serrat, Fixed Income Securities: Tools for Today’s Markets (4th ed., 2022). Ch. 14 §14.2, printed p. 353.

[2] Shreve, Stochastic Calculus for Finance II: Continuous-Time Models (2004). Ch. 1 §1.3, printed pp. 13–18.

[3] Tuckman & Serrat, Fixed Income Securities: Tools for Today’s Markets (4th ed., 2022). Ch. 14 §14.7, printed pp. 371–372, eq. 14.8 and Table 14.11.

Notation

F **Recovery par amount**. Positive currency amount promised after survival and used as the recovery base after default. It is a toy-model input and not a market price. *Units:* stated currency at the scheduled payment time.

$$F > 0$$

T **Recovery payment time**. Single future time when either the survival-state or recovered-par amount is paid. Default may occur earlier, but this toy convention delays the recovered-par payment to this time. *Units:* model-years from valuation time.

$$T > 0$$

$A(0,t)$ **Accumulation factor**. Grows one current unit over a stated future horizon under the selected rate model. *Units:* dimensionless currency-units per current currency-unit.

m **Compounding frequency**. Number of equal compounding periods per year under the stated rate convention, a positive integer fixed by the model convention. *Units:* compounding periods per year.

$D(0,t)$ **Discount factor**. Converts one deterministic future unit into value at valuation time. *Units:* current currency-units per future currency-unit.

$\mathbb{E}$ **Expectation**. Operator returning the average of a random quantity, each outcome weighted by its probability under a stated measure; a superscript names that measure when more than one is in play. *Units:* value units.

$$\mathbb{E}[X] = \int_{\Omega} X(\omega)\, d\mathbb{P}(\omega)$$

Δq_i **Interval default probability**. Probability assigned by the model to first default during one stated time interval, conditional only through the input survival curve construction. *Units:* probability between zero and one.

LGD **Loss given default**. Fraction of an explicitly stated reference amount not recovered under a deterministic recovery convention, relative to the same reference amount used by the recovery rate. *Units:* decimal fraction between zero and one.

$j^{(m)}$ **Nominal annual rate**. Annualized rate quote that must be paired with its compounding frequency. *Units:* decimal rate per year.

t_k **Payment time**. Time from the valuation date to one scheduled cash-flow event; a time coordinate, not a calendar date or payment index. *Units:* years from the valuation date.

r_m **Periodic rate**. Rate applied once in each compounding period under the stated convention, derived from the stated nominal annual quote in this model. *Units:* decimal per compounding period.

PV_0 **Present value**. Combines dated signed cash flows into one value at valuation time, using the same holder perspective as the signed cash flows. *Units:* stated currency at the valuation time.

PV_0^{RoP} **Recovery of par present value**. Present value of one maturity payment that is par after survival and a fixed fraction of par after earlier default; a positive asset value to the holder in the one-period recovery-of-par-paid-at-maturity model. *Units:* stated currency at valuation time.

R **Recovery rate**. Fraction of a stated reference amount recovered after a modeled default under an explicitly stated recovery convention; a non-negative fraction whose reference amount, payment timing, and settlement convention are set by the model that uses it. *Units:* decimal fraction between zero and one.

$\mathbb{Q}$ **Risk-neutral probability measure**. Gives model pricing weights under which discounted traded prices satisfy the martingale condition; it prices payoffs relative to a stated numeraire and is not a forecast of actual event frequencies. *Units:* dimensionless probability weights between zero and one.

CF_k **Signed cash flow**. Amount received or paid at one event from the stated holder perspective; positive means received and negative means paid by that holder. *Units:* stated currency units.

$S(0,t)$ **Survival probability**. Probability, under the explicitly stated model measure, that no modeled default has occurred between valuation time and a stated future time. *Units:* probability between zero and one.

0 **Valuation time**. Common origin from which later model times and present values are measured. *Units:* years from the valuation date.

Answers to the knowledge checks

Knowledge check 6.2.1: Recovery and risky present value

1. **(a)** A recovery cash flow equal to 40% of par is paid at maturity. Under this narrow model, recovery uses par as its base and is paid at the common maturity date.

2. **(a)** USD 500 at maturity. Recovery is 0.25 × USD 2,000 par = USD 500; the current price is not the recovery base in this model.

3. 0.65 (accepted within $\pm 1e-12$). Loss given default is 1 - recovery rate = 1 - 0.35 = 0.65.

4. 0.4 (accepted within $\pm 1e-12$). The recovery rate is 900 / 1,500 = 0.60, so loss given default is 1 - 0.60 = 0.40.

5. 893 (accepted within ±0.01). The probability-weighted maturity payoff is 1,000 × [0.90 + (1 - 0.90) × 0.40] = 940. Discounting gives 0.95 × 940 = 893.

6. 2198.8 (accepted within ±0.01). The probability-weighted payoff factor is 1 - 0.08 × 0.55 = 0.956. Present value is 2,500 × 0.92 × 0.956 = 2,198.80.