Chapter 7.1

CDS premium and protection legs

From *Credit Products Playground*, with changes after version 0.4.0, not yet released. Interactive version: https://danieltuzes.github.io/credit-and-derivatives/cds/premium-protection-legs-and-par-spread/

7.1.1 What a CDS is

A credit default swap (CDS) is a bilateral derivative on a named reference entity. The protection buyer pays a running premium to the protection seller. If a covered credit event occurs before maturity, the seller makes a loss-related protection payment. Unlike buying the entity's bond, entering a CDS does not itself lend principal to the reference entity. Tuckman and Serrat describe the fee leg and the contingent leg (in this course, the premium leg and the protection leg) in Chapter 14[1]; Hull describes the same cash-flow directions for buyer and seller, and the quarterly premium convention[2].

This lesson reports both leg present values as positive magnitudes. From the protection buyer's perspective, the net present value is the protection-leg present value minus the premium-leg present value:

$$PV_0^{\text{buyer}} = PV_0^{\text{prot}} - PV_0^{\text{prem}} \tag{7.1.1}$$

The seller has the opposite signed value in this two-party model.

7.1.2 Start from disjoint events

Let the random default-time be τ. Use the local cds-period-index i across the number-of-cds-periods n. The schedule has dates $0 = t_0 < t_1 < \cdots < t_n = T$. Define the disjoint interval events

$$G_i = \{t_{i-1} < \tau \le t_i\}, \qquad i = 1, \dots, n \tag{7.1.2}$$

and add $G_0 = \{\tau > T\}$ for survival through maturity. These events are disjoint and exhaustive under the one-default assumption.

For a local cds-event-payoff Y that is constant at y_i on each event, the risk-neutral expectation is:

$$\mathbb{E}^{\mathbb{Q}}[Y] = \sum_{i=0}^{n} y_i \, \mathbb{Q}(G_i) \tag{7.1.3}$$

This is the expectation from disjoint events of the probability lesson, with risk-neutral pricing weights. If Y is not constant within an interval, the expectation uses the conditional means:

$$\mathbb{E}^{\mathbb{Q}}[Y] = \sum_{i=0}^{n} \mathbb{Q}(G_i)\, \mathbb{E}^{\mathbb{Q}}[Y \mid G_i] \tag{7.1.4}$$

For this reason, one payment at the end of the period is not an exact treatment of a default in the middle of the period. If every interval is divided into smaller default-time intervals, the limit of the sum is an integral. Shreve gives the finite disjoint-event expectation rule used here[3]. Tuckman and Serrat's CDS table also weights cash flows by survival and default events[4].

7.1.3 The hazard rate gives the probabilities in the formulas

Let $\lambda(u)$ be the risk-neutral default intensity at time u, given survival to u. It is a deterministic function of time, measured per model-year under the risk-neutral-probability-measure $\mathbb{Q}$. It generalizes the constant Constant hazard rate λ of this lesson. Its meaning is the conditional intensity of the credit lesson, but it need not be constant: $\lambda(u)$ is the proportional rate at which the survival probability decreases at u, given survival to u. As a differential equation, this meaning is:

$$-\,dS^{\mathbb{Q}}(0,u) = \lambda(u)\, S^{\mathbb{Q}}(0,u)\, du, \qquad S^{\mathbb{Q}}(0,0) = 1 \tag{7.1.5}$$

Dividing by the (positive) survival probability turns the left side into a derivative of its logarithm, $-d\ln\left(S^{\mathbb{Q}}(0,u)\right) = \lambda(u)\, du$. Integrating from 0 to t and using $\ln\left(S^{\mathbb{Q}}(0,0)\right) = \ln(1) = 0$ gives the survival probability:

$$S^{\mathbb{Q}}(0,t) = \exp\left(-\int_0^t \lambda(u)\ du\right) \tag{7.1.6}$$

Integrating the same differential equation over one period, not from 0, gives the interval default probability for a general hazard rate:

$$\Delta q_i^{\mathbb{Q}} = \int_{t_{i-1}}^{t_i} \lambda(u)\, S^{\mathbb{Q}}(0,u)\, du = S^{\mathbb{Q}}(0,t_{i-1}) - S^{\mathbb{Q}}(0,t_i) \tag{7.1.7}$$

The constant Constant hazard rate λ of this lesson is the special case $\lambda(u) \equiv \lambda$. Then $\int_0^t \lambda(u)\, du = \lambda t$, and the two results above become:

$$S^{\mathbb{Q}}(0,t) = \exp(-\lambda t), \qquad \Delta q_i^{\mathbb{Q}} = \exp(-\lambda t_{i-1}) - \exp(-\lambda t_i) \tag{7.1.8}$$

The next two sections, on the premium leg and on the protection leg, use the general $\lambda(u)$. The section on closed forms after them uses the constant λ.

These probabilities are pricing probabilities, not forecasts, for a constant and for a general hazard rate. A hazard rate implied from CDS prices can be a risk-neutral quantity rather than a prediction of realized defaults[5]. The constant-hazard survival relation is given in §14.6 and Appendix A14.1[6].

7.1.4 Default in the middle of a period

Suppose default occurs at u in $(t_{i-1}, t_i]$. The input recovery-rate is R.

- The scheduled premium at t_i is not paid, because the reference entity did not survive to that date.
- The buyer pays premium accrued from t_{i-1} to u. In this model its year fraction is $a_i(u) = u - t_{i-1}$.
- The seller pays protection of $\text{LGD}N = (1 - R)N$.
- Both amounts triggered by default are discounted from the modeled default time u, not from t_i.

Hull notes that a final accrued premium is paid when default occurs between scheduled dates[2]. The half-period and midpoint calculations in Hull and in Tuckman and Serrat are discrete approximations[7][8]. This lesson integrates over the modeled default time exactly, for a general hazard rate and a general discount curve. The closed forms later in the lesson use a constant hazard rate and a constant interest rate.

7.1.5 Exact premium-leg expectation in this model

Let $D(0, u)$ be the general dated discount curve of the rates lesson; this section does not assume a flat rate. The cds-scheduled-premium-annuity is:

$$A_0^{\text{sched}} = \sum_{i=1}^{n} \alpha_i D(0, t_i) S^{\mathbb{Q}}(0, t_i) \tag{7.1.9}$$

The exact cds-accrued-premium-annuity is:

$$A_0^{\text{accr}} = \sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} a_i(u) D(0, u) \lambda(u) S^{\mathbb{Q}}(0, u) \, \mathrm{d}u \tag{7.1.10}$$

The cds-premium-annuity is the sum of the two annuities. The premium-leg present value is the annual spread, as a decimal, multiplied by the notional and by the premium annuity:

$$A_0^{\text{prem}} = A_0^{\text{sched}} + A_0^{\text{accr}}, \qquad PV_0^{\text{prem}} = sNA_0^{\text{prem}} \tag{7.1.11}$$

A spread quoted in basis-point per year is divided by 10,000 before it is used in (7.1.11).

7.1.6 Exact protection-leg expectation in this model

The protection-leg present value, as a positive magnitude, is:

$$PV_0^{\text{prot}} = N\text{LGD} \sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} D(0, u) \lambda(u) S^{\mathbb{Q}}(0, u) \, \mathrm{d}u \tag{7.1.12}$$

Equation (7.1.12) is an expectation. Each possible payoff at a default time is discounted and multiplied by the risk-neutral probability density of that default time. The integral then adds over all default times.

7.1.7 Closed forms and the zero-upfront par spread

This section uses a constant Constant hazard rate, $\lambda(u) \equiv \lambda$, which is a special case of the cds-general-hazard-rate. It also uses a constant continuously-compounded-risk-free-rate r. So the general discount curve of the last two sections is $D(0,u) = \exp(-ru)$.

For the regular model-year schedule, $\alpha_i = t_i - t_{i-1}$ and the maturity-time is $T = t_n$. Define the abbreviation $\kappa = r + \lambda$. Then the scheduled-premium annuity and the accrued-premium annuity are:

$$A_0^{\text{sched}} = \sum_{i=1}^{n} \alpha_i \exp(-\kappa t_i) \tag{7.1.13}$$

$$A_0^{\text{accr}} = \sum_{i=1}^{n} \lambda \exp(-\kappa t_{i-1}) \frac{1 - \exp(-\kappa\alpha_i)(1 + \kappa\alpha_i)}{\kappa^2} \tag{7.1.14}$$

The cds-protection-factor, per unit of notional, is:

$$B_0^{\text{prot}} = \text{LGD}\, \lambda \frac{1 - \exp(-\kappa T)}{\kappa} \tag{7.1.15}$$

At $\kappa = 0$, the continuous limits replace the two fractions by $\alpha_i^2/2$ and T, respectively. The implementation uses numerically stable forms of those limits.

The cds-par-spread $s^\star$ is the running spread that makes the legs equal with no upfront amount:

$$s^\star = \frac{B_0^{\text{prot}}}{A_0^{\text{prem}}}, \qquad PV_0^{\text{prem}}(s^\star) = PV_0^{\text{prot}} \tag{7.1.16}$$

Tuckman and Serrat state the condition that the fair spread makes the two legs equal, and the relationship between spread and upfront[9]. The next lesson distinguishes this model par spread from a standard running coupon and a signed upfront amount.

Example 7.1.1: Exact default-time CDS legs

A finite event partition

Suppose a two-period protection payoff, simplified to one value per event, is USD 6,000 on G_1, USD 5,800 on G_2, and zero on the survival event G_0. The risk-neutral probabilities of G_1, G_2, and G_0 are 0.02, 0.03, and 0.95. The expected payoff before discounting is:

$$0(0.95) + 6{,}000(0.02) + 5{,}800(0.03) = \text{USD } 294$$

This finite sum is exact only because the payoff is constant on each event. In an exact CDS valuation, the discounted payoff varies with the default time inside an event. So the exact valuation uses an integral over all default times.

One-year exact legs

Take notional $N = \text{USD } 1{,}000{,}000$, running spread $s = 0.01$ per year (100 bp/year), recovery $R = 0.40$, constant risk-neutral hazard $\lambda = 0.02$ per model-year, continuously compounded risk-free rate $r = 0.04$ per model-year, two equal periods, and maturity $T = 1$ model-year.
The tested domain code gives:

Factor	Value
Scheduled premium annuity	0.956105034 model-years
Accrued-premium annuity	0.004828691 model-years
Premium annuity	0.960933725 model-years
Protection factor	0.011647093 per unit notional

The premium-leg present value is:

$$PV_0^{\text{prem}} = 0.01(1{,}000{,}000)(0.960933725) \approx \text{USD } 9{,}609.34$$

The protection-leg present value is:

$$PV_0^{\text{prot}} = 1{,}000{,}000(0.011647093) \approx \text{USD } 11{,}647.09$$

The present value of the accrued premium is USD 48.29. It is small in this example. The model calculates it at every possible default time; it does not approximate it by one half of a period.

Solve the exact par spread

With the same inputs, the par spread is:

$$s^\star = \frac{0.011647093}{0.960933725} \approx 0.012120600 = 121.206 \text{ bp/year}$$

At the unrounded par spread, the present value of each leg is approximately USD 11,647.09. The notional cancels in the par spread, because both legs are proportional to the same notional.

7.1.8 Generalizing the premium leg: an extinguishing swap

This lesson uses the name **extinguishing swap** for the construction below. The name belongs to this course; it is not a standard market term or textbook term. An extinguishing swap is any schedule of cash flows that stops permanently at the default of the reference entity. The premium leg above has this property.

Consider the same dates $0 = t_0 < t_1 < \cdots < t_n = T$ and the same reference entity. Replace the scheduled premium $sN\alpha_i$ at each date with an arbitrary deterministic, signed net cash flow c_i for one named perspective. Under the signed cash-flow convention of this course, a positive c_i is received and a negative c_i is paid. As with the premium leg, a scheduled amount is paid only on survival to its date. Every later scheduled amount is extinguished at the default time. In this lesson, c_i is an input and deterministic; it does not depend on a floating rate. If a cash flow accrues continuously between dates, an accrued-at-default term can be added with the same construction as the cds-accrued-premium-annuity. This lesson does not repeat that term.

The expectation from disjoint events, used for both CDS legs above, gives the cds-extinguishing-swap-present-value of the scheduled cash flows. It is the same survival-weighted discounting that gave A_0^{sched}:

$$PV_0^{\text{ext}} = \sum_{i=1}^{n} c_i \, D(0,t_i) \, S^{\mathbb{Q}}(0,t_i) \tag{7.1.17}$$

With $c_i = sN\alpha_i$ for every i, the present value is sNA_0^{sched}, the scheduled-payment term of the premium leg. So the premium leg is the special case of an extinguishing swap whose cash flows are one running spread paid on a fixed notional.

Example 7.1.2: A two-date extinguishing swap

A two-date extinguishing swap

Use the model of the "One-year exact legs" example: $\kappa = r + \lambda = 0.06$ per model-year, with semiannual dates $t_1 = 0.5$ and $t_2 = 1$. The discounted-survival factors are:

$$D(0,t_1)S(0,t_1) = \exp(-0.06\times 0.5) \approx 0.970445534, \qquad D(0,t_2)S(0,t_2) = \exp(-0.06\times 1) \approx 0.941764534$$

Suppose this extinguishing swap's schedule is a USD 30,000 receipt at t_1 and a USD 12,000 payment at t_2, so $c_1 =$ USD 30,000 and $c_2 = -$USD 12,000. The present value of the extinguishing swap is:

$$PV_0^{\text{ext}} = 30{,}000(0.970445534) + (-12{,}000)(0.941764534) \approx \text{USD } 17{,}812.19$$

Both terms use the same $D(0,t_i)S(0,t_i)$ factors as the premium leg. Only the signed cash flows c_i are different from the premium leg's $sN\alpha_i$.

7.1.9 Why there is no volatility input here

Risk-neutral valuation does not remove volatility. A change from the real-world measure to $\mathbb{Q}$ changes the pricing probabilities, and in common diffusion models it changes the drift. It does not change the size of the random fluctuations. Shreve shows that the Brownian volatility term remains after the risk-neutral change of measure[10].

This CDS model needs no separate volatility parameter for two reasons. First, its risk-free rate, hazard rate, and recovery rate are deterministic. Second, both legs are expectations that are linear in the cash flows. In other cases, the distributions, the dependence, and the volatility of the inputs could matter: if rates, the hazard rate, or recovery were stochastic, or if the product were a CDS option, had wrong-way risk, or had nonlinear exposure. The risk-neutral measure gives the pricing weights. The restrictive model assumptions, not the measure, remove the need for a separate volatility input.

7.1.10 Both legs for one contract

Figure 7.1.1 shows the present value of each leg for one contract in this model. The explorer integrates the accrued premium and the protection payment over the exact modeled default time.

7.1.11 Check your understanding

The assessment provides direct and transfer evidence for leg perspective, premium-leg magnitude, protection-leg magnitude, and par spread.

Knowledge check 7.1.1: CDS legs and par spread

1. Under the lesson sign convention, both CDS legs are reported as positive present-value magnitudes. Which expression gives the protection buyer's signed net value?

   (a) Protection-leg magnitude minus premium-leg magnitude

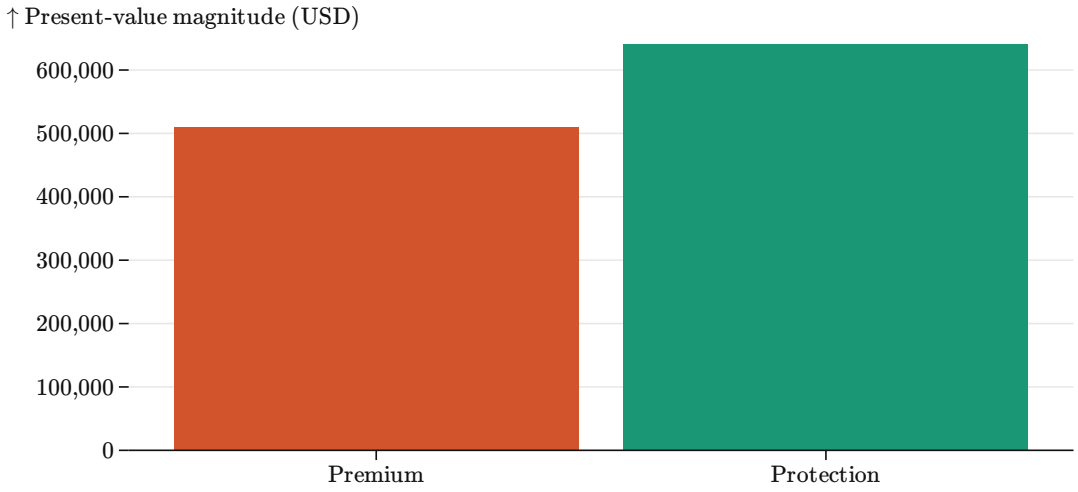


Figure 7.1.1 **CDS leg explorer.** The present values of the premium leg and the protection leg, as positive magnitudes in USD, for a notional of USD 10,000,000 and four equal model periods a year. The protection buyer's value is the protection leg minus the premium leg. Shown for a contractual spread of 120.0 bp/year, a constant risk-neutral hazard rate of 2.5%/year, a recovery rate of 40%, a continuously compounded risk-free rate of 4% and a maturity of 5 years: premium leg USD 509,703, protection leg USD 640,321.

(b) Premium-leg magnitude minus protection-leg magnitude

(c) Protection-leg magnitude plus premium-leg magnitude

2. A CDS has a positive protection-leg present-value magnitude of USD 35 and a positive premium-leg present-value magnitude of USD 28. What is the protection buyer's signed net value in USD?

   (a) +7

   (b) -7

   (c) +63

3. In the lesson's exact default-time model, a one-year annual-pay CDS has scheduled-premium annuity 0.941764533584 model-years and exact accrued-on-default annuity 0.009608857782 model-years. Calculate the positive premium-leg present-value magnitude in USD for notional USD 1,000,000 and running spread 100 basis points (0.01 per year).

4. A two-year quarterly CDS has notional USD 3,000,000 and running spread 125 basis points (0.0125 per year). Exact integration under the input constant-hazard model gives scheduled-premium annuity 1.829782217887 model-years and accrued-on-default annuity 0.011512762522 model-years. Calculate the positive premium-leg magnitude in USD.

5. For a one-year CDS, exact integration over modeled default time gives a discounted risk-neutral default-density integral of 0.019411822139. Calculate the positive protection-leg magnitude in USD for notional USD 1,000,000 and recovery rate 0.40.

6. Use the exact flat-hazard protection formula $N(1-R)\lambda[1-\exp(-(r+\lambda)T)]/(r+\lambda)$. Calculate the positive protection-leg magnitude in USD for N = USD 3,000,000, $R = 0.35$, $\lambda = 0.05$ per year, $r = 0.03$ per year, and $T = 2$ years.

7. Calculate the zero-upfront par spread in basis points per year when the exact premium annuity is 0.951373391366 model-years and the exact protection factor is 0.011647093283 per unit notional.

8. A different exact default-time model produces scheduled-premium annuity 1.829782217887 model-years, accrued-on-default annuity 0.011512762522 model-years, and protection factor 0.060066585732 per unit notional. Calculate its zero-upfront par spread in basis points per year.

Answers: *end of the chapter.*

7.1.12 Model boundary and review note

The valuation is exact only inside this simplified model. The premium leg and the protection leg are derived for a general, possibly time-varying risk-neutral hazard rate and a general dated discount curve. The closed forms, the numeric examples, the explorer, and the assessment use a special case: one constant risk-neutral hazard rate, one deterministic continuously compounded risk-free rate, a deterministic recovery rate, and settlement at the default time. The extinguishing swap is a construction of this lesson that generalizes the premium leg; it is not a named market instrument. The model omits calendar schedules, day counts, business-day rules, payment delays, term-structure calibration, auctions, deliverables, credit-event legal definitions, counterparty risk, collateral, funding, restructuring, and transaction costs. All materially changed content remains ‘draft‘ pending independent human review.

References

[1] Tuckman & Serrat, Fixed Income Securities: Tools for Today’s Markets (4th ed., 2022). §14.5, printed pp. 361–362.

[2] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). §24.1, printed pp. 548–550.

[3] Shreve, Stochastic Calculus for Finance II: Continuous-Time Models (2004). Ch. 1 §1.3, printed pp. 13–18.

[4] Tuckman & Serrat, Fixed Income Securities: Tools for Today’s Markets (4th ed., 2022). §14.6, printed pp. 368–370, Table 14.10 and eqs. 14.6–14.7.

[5] Tuckman & Serrat, Fixed Income Securities: Tools for Today’s Markets (4th ed., 2022). §14.7, printed p. 371.

[6] Tuckman & Serrat, Fixed Income Securities: Tools for Today’s Markets (4th ed., 2022). printed pp. 367–368 and 505, eqs. 14.4–14.5 and A14.1–A14.2.

[7] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). §24.2, printed pp. 551–554, Tables 24.1–24.4.

[8] Tuckman & Serrat, Fixed Income Securities: Tools for Today’s Markets (4th ed., 2022). §14.6, printed pp. 368–370, Table 14.10.

[9] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). §14.6, printed pp. 370–371, and Appendix A14.2, printed pp. 506–507.

[10] Shreve, Stochastic Calculus for Finance II: Continuous-Time Models (2004). Ch. 5 §5.2.2, printed pp. 214–217.

Notation

A_0^{accr} **CDS accrued premium annuity**. Present-value factor for exact premium accrual paid at modeled default. It integrates accrued year fraction, discounting, and risk-neutral default density over every possible default time. *Units:* model-years.

$a_i(u)$ **CDS accrued year fraction at default**. Year fraction accrued from the preceding premium date to a possible default time in period i. In the regular model-year schedule it equals elapsed model time since the period start. *Units:* model-years of premium accrual.

$$a_i(u) = u - t_{i-1}$$

κ **CDS combined decay rate**. Abbreviation for the sum of the deterministic risk-free rate and constant hazard rate. It shortens the closed forms and has no separate market interpretation. *Units:* decimal rate per model-year.

$$\kappa = r + \lambda$$

G_i **CDS default event**. Event that modeled default occurs after one premium date and no later than the next. Together with survival through maturity, these events are disjoint and exhaustive in the one-default model.

$$G_i = \{t_{i-1} < \tau \leq t_i\}$$

u **CDS default integration time**. Integration variable ranging over every possible default time inside one CDS period, or over the interval used to cumulate a time-varying hazard rate into a survival probability. It is a deterministic dummy variable; the random default time is a different symbol. *Units:* model-years after valuation time.

Y **CDS event payoff**. Random discounted payoff used to connect event partitions to a CDS-leg expectation. Its currency sign and perspective must be stated before taking an expectation. *Units:* stated currency at valuation time.

y_i **CDS event payoff value**. Constant value taken by the generic payoff on one event in a finite partition. If the payoff varies inside an event, conditional expectation replaces this constant. *Units:* stated currency at valuation time.

c_i **CDS extinguishing swap cash flow**. Deterministic signed net cash flow scheduled at one payment date for one named perspective, in the extinguishing-swap construction that generalizes the premium leg to an arbitrary schedule. Positive means received, under the signed cash-flow convention used throughout this course. *Units:* stated currency at the scheduled payment time.

PV_0^{ext} **CDS extinguishing swap present value**. Present value, for the named perspective, of a scheduled net cash-flow stream that stops entirely once the reference entity defaults. This is the extinguishing-swap construction generalizing the premium leg. *Units:* stated currency at valuation time.

$\lambda(u)$ **CDS general hazard rate**. General risk-neutral default intensity, written as a function of time rather than assumed constant. It generalizes the constant hazard rate used elsewhere in this course to a deterministic function of time; the constant case is recovered by setting it equal to one fixed value for every time. *Units:* decimal intensity per model-year.

i **CDS period index**. Selects one default interval and its following scheduled premium date. The index is bookkeeping and is not itself a time, probability, or currency amount.

$$i \in \{1, \ldots, n\}$$

B_0^{prot} **CDS protection factor**. Present value of protection per unit CDS notional in the simplified model. It includes loss given default, discounting, and the risk-neutral default-time density. *Units:* present currency-units per notional currency-unit.

A_0^{sched} **CDS scheduled premium annuity**. Present-value factor for scheduled premiums per unit notional and unit annual running rate. It includes a scheduled payment only when the name survives to that payment date. *Units:* model-years.

r **Continuously compounded risk-free rate**. Deterministic annual decimal rate used to discount this lesson's model cash flows. It is a pricing input and is separate from the hazard rate. *Units:* decimal rate per model-year.

$$D(0, u) = \exp(-ru)$$

n **Number of CDS periods**. Counts the finite premium and default intervals in the simplified CDS schedule. It is a positive integer determined by model term and payment frequency.

$$n \in \{1, 2, \ldots\}$$

$A(0, t)$ **Accumulation factor**. Grows one current unit over a stated future horizon under the selected rate model. *Units:* dimensionless currency-units per current currency-unit.

1 bp **Basis point**. Rate-change unit equal to one hundredth of one percentage point. *Units:* decimal rate.

m_{B} **Bond payment frequency**. Number of scheduled coupon payments per year in the simplified regular bond, a positive integer. *Units:* scheduled coupon payments per year.

α_i **CDS accrual year fraction**. Input year fraction that converts an annualized spread into the premium amount for one scheduled period; a positive model input. *Units:* model-years of premium accrual.

s **CDS contract spread**. Annualized premium rate applied to notional and each stated accrual year fraction; a positive rate paid by the protection buyer in the simplified lesson. *Units:* decimal per year in calculations; basis points per year when explicitly quoted.

N **CDS notional**. Reference currency amount that scales the simplified premium and protection legs; a positive amount, not itself a signed leg cash flow. *Units:* stated currency.

$s^\star$ **CDS par spread**. Contractual spread that makes the two positive leg magnitudes equal at valuation time with zero upfront amount, solved under the lesson's input curves, recovery, timing, and accrued-premium convention. *Units:* decimal per year in calculations; basis points per year when explicitly quoted.

A_0^{prem} **CDS premium annuity**. Present-value coefficient that multiplies contractual spread and notional in the simplified premium leg; positive, and includes scheduled premiums and exact accrued premium under the lesson's default-time model. *Units:* model-years of present value per unit notional.

PV_0^{prem} **CDS premium leg present value**. Positive valuation-time magnitude of the simplified protection buyer's premium payments; its signed contribution to protection-buyer net value is negative. *Units:* stated currency at valuation time.

PV_0^{buyer} **CDS protection buyer net present value**. Signed protection-buyer value equal to protection-leg magnitude minus premium-leg magnitude; positive favors the protection buyer and negative favors the protection seller in the two-leg toy model. *Units:* stated currency at valuation time.

PV_0^{prot} **CDS protection leg present value**. Positive valuation-time magnitude of the simplified loss-given-default payment received by the protection buyer after a modeled default. *Units:* stated currency at valuation time.

m **Compounding frequency**. Number of equal compounding periods per year under the stated rate convention, a positive integer fixed by the model convention. *Units:* compounding periods per year.

τ **Default time**. Random model time at which the reference entity first defaults. It is measured in model-years from valuation time under the stated risk-neutral model. *Units:* model-years after valuation time.

$$\tau > 0$$

$D(0,t)$ **Discount factor**. Converts one deterministic future unit into value at valuation time. *Units:* current currency-units per future currency-unit.

$\mathbb{E}$ **Expectation**. Operator returning the average of a random quantity, each outcome weighted by its probability under a stated measure; a superscript names that measure when more than one is in play. *Units:* value units.

$$\mathbb{E}[X] = \int_{\Omega} X(\omega)\, d\mathbb{P}(\omega)$$

λ **Constant hazard rate**. Constant conditional default intensity used by the lesson's simplified exponential survival model; a risk-neutral pricing input conditional on survival to the current instant, not a cumulative probability. *Units:* decimal intensity per model-year.

Δq_i **Interval default probability**. Probability assigned by the model to first default during one stated time interval, conditional only through the input survival curve construction. *Units:* probability between zero and one.

LGD **Loss given default**. Fraction of an explicitly stated reference amount not recovered under a deterministic recovery convention, relative to the same reference amount used by the recovery rate. *Units:* decimal fraction between zero and one.

T **Maturity time**. Final scheduled time when principal is redeemed in the simplified bond. *Units:* model-years from the valuation date.

$j^{(m)}$ **Nominal annual rate**. Annualized rate quote that must be paired with its compounding frequency. *Units:* decimal rate per year.

t_k **Payment time**. Time from the valuation date to one scheduled cash-flow event; a time coordinate, not a calendar date or payment index. *Units:* years from the valuation date.

r_m **Periodic rate**. Rate applied once in each compounding period under the stated convention, derived from the stated nominal annual quote in this model. *Units:* decimal per compounding period.

R **Recovery rate**. Fraction of a stated reference amount recovered after a modeled default under an explicitly stated recovery convention; a non-negative fraction whose reference amount, payment timing, and settlement convention are set by the model that uses it. *Units:* decimal fraction between zero and one.

$\mathbb{Q}$ **Risk-neutral probability measure**. Gives model pricing weights under which discounted traded prices satisfy the martingale condition; it prices payoffs relative to a stated numeraire and is not a forecast of actual event frequencies. *Units:* dimensionless probability weights between zero and one.

$S(0,t)$ **Survival probability**. Probability, under the explicitly stated model measure, that no modeled default has occurred between valuation time and a stated future time. *Units:* probability between zero and one.

t **Time from valuation**. A time measured in years from the valuation time; a time coordinate, not a calendar date or a schedule index. With a row index it is a payment time. *Units:* years from the valuation date.

0 **Valuation time**. Common origin from which later model times and present values are measured. *Units:* years from the valuation date.

Answers to the knowledge checks

Knowledge check 7.1.1: CDS legs and par spread

1. **(a)** Protection-leg magnitude minus premium-leg magnitude. The protection leg is an inflow and the premium leg is an outflow for the protection buyer, so signed net value is protection minus premium.

2. **(a)** +7. From the protection buyer's perspective, net value is 35 - 28 = +7.

3. 9513.73391366 (accepted within ±0.01). The exact total premium annuity is 0.941764533584 + 0.009608857782 = 0.951373391366 model-years. The positive premium magnitude is 0.01 × 1,000,000 × 0.951373391366 = USD 9,513.73391366.

4. 69048.56176534 (accepted within ±0.01). The total exact premium annuity is 1.841294980409 model-years. Multiplying 0.0125 × 3,000,000 × 1.841294980409 gives USD 69,048.56176534. The accrued factor comes from integrating default time, not a half-period replacement.

5. 11647.0932834 (accepted within ±0.01). Loss given default is 1 - 0.40 = 0.60. The exact protection magnitude is 1,000,000 × 0.60 × 0.019411822139 = USD 11,647.0932834.

6. 180199.757197 (accepted within ± 0.01). Here $r + \lambda = 0.08$. Substitution gives 3,000,000 $\times$ 0.65 $\times$ 0.05 $\times$ [1 - exp(-0.16)] / 0.08 = USD 180,199.757197. This discounts over actual modeled default time through the integral's closed form.

7. 122.423996602 (accepted within ± 0.001). The par annual decimal spread is 0.011647093283 / 0.951373391366 = 0.0122423996602. Multiplying by 10,000 gives 122.423996602 basis points per year; notional cancels.

8. 326.219244453 (accepted within ± 0.001). The exact total premium annuity is 1.841294980409. The par spread is 0.060066585732 / 1.841294980409 = 0.0326219244453 per year, or 326.219244453 basis points per year.