Chapter 7.2

CDS market-standard quote and upfront

From *Credit Products Playground*, with changes after version 0.4.0, not yet released. Interactive version: https://danieltuzes.github.io/credit-and-derivatives/cds/market-standard-quote-and-upfront/

7.2.1 What you will be able to do

After this lesson, you should be able to:

- distinguish a conventional spread quote from the contract's fixed running coupon;
- identify which quantity is an input and which is solved in each quotation direction;
- calculate the signed upfront amount that makes protection-buyer inception value zero;
- reverse the simplified conversion without incorrectly freezing a quote-dependent premium annuity.

7.2.2 Three quantities with three different roles

Standardized CDS quotation separates a fixed running coupon from a market-determined upfront amount. Textbook treatments describe how to convert a quoted spread, treated as a par spread, into the value of a contract with a fixed coupon. The official converter specification accepts either the spread or the upfront and returns the other. [1] [2] [3]

This lesson uses the name **market-standard quote** (**MSQ**) for this conventional quoted spread. The name belongs to this course. The books and the official specification more commonly say *quoted spread*, *conventional spread*, or *spread*. This lesson does not claim that "MSQ" is a universal abbreviation.

The three quantities have different roles, but they are not three independent inputs:

- the cds-market-standard-quote s_{MSQ} is treated as a zero-upfront cds-par-spread and calibrates this lesson's flat Constant hazard rate λ;
- the cds-standard-coupon c_{std} determines the running premium cash flows;
- the cds-upfront-amount U_0 offsets the difference in value at inception.

The phrase "upfront fee" is also common. This lesson uses **upfront amount**, because the amount is signed. The sign of U_0 means:

- $U_0 > 0$: the protection buyer pays;
- $U_0 = 0$: no upfront cash changes hands in the toy model;
- $U_0 < 0$: the protection buyer receives $-U_0$.

In the formulas, rates are decimals per year. Convert a basis-point quote to decimal form by dividing by 10,000.

7.2.3 First decide which quantities are inputs and which are solved

The equation that connects the par spread and the hazard rate can be used in two directions. So a quantity can be an input in one calculation and solved in another. Table 7.2.1 shows the inputs and the solved quantities of each calculation.

Table 7.2.1 **Inputs and solved quantities of each calculation.** For the model par spread and for each direction of the quote conversion, the quantities taken as inputs and the quantities solved or calculated.

Calculation	Input	Solved or calculated
Model par spread	Hazard rate, recovery, discount input, maturity, and cash-flow conventions	Par spread $s^\star$
MSQ to upfront	Market quote s_{MSQ}, standard coupon c_{std}, and the converter conventions	Flat hazard rate λ, then signed upfront U_0
Upfront to MSQ	Market upfront U_0, standard coupon c_{std}, and the converter conventions	Flat hazard rate λ, then s_{MSQ}

If the hazard rate is an input, the par spread is the output:

$$s^\star = \frac{PV_0^{\text{prot}}(\lambda)}{NA_0^{\text{prem}}(\lambda)} \tag{7.2.1}$$

If the MSQ is the input instead, (7.2.1) is a calibration equation: the quoted spread is the input, and the flat hazard rate is the unknown. If the upfront amount is the input, the upfront balance is the calibration equation. The market participant or the price source provides the market level. The standard converter specifies how the spread and the upfront are converted into each other under common assumptions. [4]

So do not fix both s_{MSQ} and λ independently. When the other model inputs are fixed, s_{MSQ} and λ must together satisfy the zero-upfront condition.

7.2.4 Why one upfront amount is enough at inception

For fixed model inputs, one positive cds-protection-leg-present-value summarizes all future protection cash flows. One positive cds-premium-leg-present-value summarizes all fixed-coupon cash flows. The difference of the two present values is one currency amount. So one cash transfer at time zero can offset that difference exactly. The upfront balance is:

$$PV_0^{\text{prot}} = PV_0^{\text{prem}} + U_0 \tag{7.2.2}$$

Equivalently, the value to the protection buyer at inception, after the signed upfront amount, is zero:

$$0 = PV_0^{\text{prot}} - PV_0^{\text{prem}} - U_0 \tag{7.2.3}$$

This is a value identity, not a claim that the CDS becomes riskless. After inception its value can change when credit, interest-rate, recovery, liquidity, or other inputs change.

7.2.5 Risk-neutral does not mean risk-free

The calibration uses **risk-neutral pricing probabilities**. They are not necessarily real-world default forecasts. Under risk-neutral valuation, probabilities are pricing weights, chosen consistently with traded prices and risk-free discounting. "Risk-free" describes the discount input, not the probability measure. So the hazard rate inferred from prices can differ from an estimated real-world default intensity. [5] [6]

A change of probability measure does not remove volatility. This converter has no separate volatility input, for two reasons. Its discount rate, hazard rate, recovery rate, and schedule are deterministic. Its leg values are expectations that are linear in the one-default cash flows. A model with stochastic rates, stochastic default intensity, stochastic recovery, wrong-way dependence, or CDS options can require volatility and correlation inputs. The risk-neutral measure gives pricing weights; it does not remove those risks.

7.2.6 Convert MSQ to signed upfront

The conversion must use one internally consistent model. For each candidate constant hazard rate, calculate the positive cds-premium-annuity $A_0^{\text{prem}}(\lambda)$ and the protection-leg present value. First, solve the zero-upfront quote condition for λ:

$$s_{\text{MSQ}} N A_0^{\text{prem}}(\lambda) = PV_0^{\text{prot}}(\lambda) \tag{7.2.4}$$

The root λ of (7.2.4) is the flat risk-neutral hazard rate implied by the MSQ, under the assumptions of this lesson on the schedule, discounting, recovery, and accrual at default. The premium-leg present value with the fixed coupon is then:

$$PV_0^{\text{prem}}(c_{\text{std}}; \lambda) = c_{\text{std}} N A_0^{\text{prem}}(\lambda) \tag{7.2.5}$$

Substituting (7.2.4) and (7.2.5) into the upfront balance (7.2.2) gives the upfront amount:

$$\begin{aligned} U_0 &= PV_0^{\text{prot}}(\lambda) - c_{\text{std}} N A_0^{\text{prem}}(\lambda) \\ &= N A_0^{\text{prem}}(\lambda) \left(s_{\text{MSQ}} - c_{\text{std}} \right) \end{aligned} \tag{7.2.6}$$

The substitution in (7.2.6) is valid only because every term uses the same model and the same calibrated hazard rate. The premium annuity and the notional are positive. So an MSQ above the fixed coupon gives a positive upfront amount, which the buyer pays. An MSQ below the fixed coupon gives a negative upfront amount, which the buyer receives.

7.2.7 Convert signed upfront back to MSQ

When the input is a signed upfront amount, do not rearrange the last line of (7.2.6) with the annuity held fixed. In this flat-hazard converter, both the protection-leg present value and the premium annuity change with the unknown hazard rate. Instead, solve this equation for the hazard rate:

$$U_0 = PV_0^{\text{prot}}(\lambda) - c_{\text{std}} N A_0^{\text{prem}}(\lambda) \tag{7.2.7}$$

Solve (7.2.7) for a non-negative λ. Then (7.2.8) gives the MSQ from this λ:

$$s_{\text{MSQ}} = \frac{PV_0^{\text{prot}}(\lambda)}{N A_0^{\text{prem}}(\lambda)} \tag{7.2.8}$$

If a curve and its annuity have already been calibrated and are held fixed, rearranging (7.2.6) gives a shortcut:

$$s_{\text{MSQ}} = c_{\text{std}} + \frac{U_0}{N A_0^{\text{prem}}(\lambda)} \tag{7.2.9}$$

This shortcut is not the full conversion from upfront to quote when the quote itself determines λ, because then the denominator changes with the quote.

7.2.8 When MSQ and par spread are equal

Within one internally consistent risk-neutral model, the MSQ and the par spread are equal. After the converter has inferred one λ, from the spread or from the upfront amount, the following quantities are equal:

$$\begin{aligned} s_{\text{MSQ}} &= s^\star \\ &= \frac{PV_0^{\text{prot}}(\lambda)}{N A_0^{\text{prem}}(\lambda)} \\ &= c_{\text{std}} + \frac{U_0}{N A_0^{\text{prem}}(\lambda)} \end{aligned} \tag{7.2.10}$$

The equality in (7.2.10) is a risk-neutral pricing identity. No real-world default probability enters it. The equality requires the same pricing probabilities, discount inputs, recovery, schedule, default-accrual treatment, and settlement convention in every term.

The phrase **par spread from another curve** needs more care. The observed trade with a standard coupon gives one combined price:

$$U_0 = PV_0^{\text{prot}} - c_{\text{std}} N A_0^{\text{prem}} \tag{7.2.11}$$

This price does not show the protection-leg present value and the premium annuity separately. The standard converter separates them by assuming its constant-hazard model. A different risk-neutral term structure can reprice the same contract, with its fixed coupon and upfront amount, but with a different premium annuity. That term structure then gives a different par coupon to a hypothetical zero-upfront contract. The official converter itself distinguishes its single constant hazard rate from the term structure of a fuller credit-curve model. [7]

This difference is not an arbitrage by itself, because the observed fixed-coupon CDS and the hypothetical zero-upfront CDS have different premium cash flows. No arbitrage requires identical tradable cash flows to have the same price. But suppose a zero-upfront CDS that pays s_{MSQ} could

be traded on otherwise identical terms. Then a curve calibrated consistently to that traded contract would have to give it zero value. [8]

Example 7.2.1: Quote and upfront conversions

MSQ to buyer-paid upfront

Consider a five-year CDS with quarterly premiums and notional N = USD 10,000,000, 2% continuously compounded risk-free rate, 40% recovery, MSQ $s_{\text{MSQ}} = 0.015$ per year (150 basis points per year), and fixed coupon $c_{\text{std}} = 0.010$ per year (100 basis points per year).
Under the flat-hazard model of this lesson, with exact default times, the calibration to the quote gives:

$$\lambda = 0.024937617 \text{ per year}, \qquad A_0^{\text{prem}}(\lambda) = 4.466915290 \text{ model-years}$$

Because the calibrated quote is a par spread, the protection-leg present value is:

$$PV_0^{\text{prot}} = 0.015(10{,}000{,}000)(4.466915290) = \text{USD } 670{,}037.29$$

The premium-leg present value with the fixed coupon is:

$$PV_0^{\text{prem}} = 0.010(10{,}000{,}000)(4.466915290) = \text{USD } 446{,}691.53$$

The upfront amount is the difference of the two present values:

$$U_0 = 670{,}037.29 - 446{,}691.53 = \text{USD } 223{,}345.76$$

The upfront amount is positive, so the protection buyer pays it. As a check, $670{,}037.29 - 446{,}691.53 - 223{,}345.76 = 0$, up to the rounding of the displayed values.

Why reverse conversion is a solve

Suppose the input for the same contract is the signed upfront amount U_0 = USD 223,345.76, and the MSQ is unknown. A solver changes the flat λ and recalculates both the protection-leg present value and the premium annuity, until (7.2.7) holds.
The root is approximately $\lambda = 0.024937617$ per year. Only with this root does (7.2.8) give $s_{\text{MSQ}} \approx 0.015$ (150 basis points per year).
The shortcut with the annuity fixed at 4.466915290 gives the same answer here, but only because this annuity was already calculated at the solution's hazard rate. Before the solve, this annuity is unknown.

7.2.9 What a real standard converter adds

The official converter uses dates and market conventions. Its inputs include the trade date and the maturity date, the notional, the standard coupon, the recovery rate, the discount inputs, and either the spread or the upfront. Its outputs include the other quotation and a cash-settlement amount. [3]

This lesson is not an implementation of that standard model. It uses model-year periods and a single flat hazard rate. A production conversion must also handle calendar schedules, day-count and business-day conventions, curve construction and interpolation, settlement timing, the accrued amount, and other contract details. The identities of this lesson explain why the quote, the coupon, and the upfront amount differ. But the numbers in this lesson are not cash-settlement instructions for a trade.

7.2.10 Check your understanding

The assessment separates quote interpretation, signed upfront arithmetic, and the reverse flat-hazard solve. Each competency has direct and transfer evidence.

Knowledge check 7.2.1: CDS market-standard quote and upfront

1. This lesson uses MSQ as a local abbreviation for market-standard quote. Which statement matches its simplified conversion model?

   (a) MSQ is the conventional quoted spread treated as a zero-upfront par spread for calibrating the flat hazard rate

   (b) MSQ is always the fixed running coupon paid on the contract

   (c) MSQ is the signed upfront cash amount divided by notional

2. A simplified converter is given an MSQ of 180 basis points per year and a standard coupon of 100 basis points per year. Which rate determines the scheduled running premium cash flows?

   (a) The 100-basis-point standard coupon

   (b) The 180-basis-point MSQ

   (c) The 80-basis-point difference

3. Calculate the signed upfront amount in USD when notional is USD 10,000,000, the quote-implied premium annuity is 4.0 model-years, MSQ is 150 basis points per year, and the fixed coupon is 100 basis points per year. Positive means paid by the protection buyer.

4. Calculate the signed upfront amount in USD when notional is USD 5,000,000, the quote-implied premium annuity is 2.5 model-years, MSQ is 300 basis points per year, and the fixed coupon is 500 basis points per year. Positive means paid by the protection buyer; enter a negative number if the buyer receives cash.

5. For a curve that is already calibrated and held fixed, calculate the market-standard quote in basis points per year from notional USD 8,000,000, premium annuity 4.0 model-years, fixed coupon 100 basis points per year, and signed upfront USD 160,000 paid by the protection buyer.

6. Use the lesson's one-year flat-hazard model with one annual period, zero continuously compounded risk-free rate, recovery 40%, notional USD 10,000,000, and fixed coupon 1% per year. For hazard rate lambda, A(lambda) = (1 - exp(-lambda))/lambda and the par quote is (1 - 0.40)lambda. A signed upfront of USD 195,082.302 is paid by the protection buyer. Root solve U0/N = A(lambda)[(1 - 0.40)lambda - 0.01], then report the quote in basis points per year.

Answers: *end of the chapter.*

7.2.11 Model boundary and review note

This AI-assisted lesson and all cited locators remain `draft` pending human editorial and quantitative review. All input values are invented for teaching. The converter assumes one deterministic discount rate, one deterministic recovery rate, one constant risk-neutral hazard rate, regular model-year periods, exact default-time accrued premium inside that toy schedule, and an upfront payment at time zero. It omits the contract details and numerical methods listed above, as well as stochastic rates, stochastic intensity or recovery, volatility and correlation calibration, counterparty risk, collateral, funding, liquidity, and transaction costs.

References

[1] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.6, printed pp. 366-371; Appendix A14.2, printed pp. 506-507, eqs. A14.5-A14.7.

[2] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.4, printed pp. 556-557.

[3] International Swaps and Derivatives Association, ISDA CDS Standard Model and Standard CDS Contract Converter Specification. Standard CDS Contract Converter Specification, version May 5, 2009, Functionality and Specification, printed p. 1. https://www.cdsmodel.com/documentation.html

[4] International Swaps and Derivatives Association, ISDA CDS Standard Model and Standard CDS Contract Converter Specification. Standard CDS Contract Converter Specification, version May 5, 2009, Functionality and Specification, printed pp. 1 and 4. https://www.cdsmodel.com/documentation.html

[5] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 7 §7.3, printed pp. 182-184, eqs. 7.7-7.8; Ch. 14 §14.7, printed p. 371.

[6] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 23 §23.5, printed pp. 528-530.

[7] International Swaps and Derivatives Association, ISDA CDS Standard Model and Standard CDS Contract Converter Specification. Standard CDS Examples, version October 26, 2012, printed p. 2 (the converter uses a single flat hazard rate rather than a term structure of flat spreads). https://www.cdsmodel.com/documentation.html

[8] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 1 §§1.3-1.4, printed pp. 53-57 (law of one price, replication, and arbitrage enforcement).

Notation

$A(0,t)$ **Accumulation factor**. Grows one current unit over a stated future horizon under the selected rate model. *Units:* dimensionless currency-units per current currency-unit.

$1\,\mathrm{bp}$ **Basis point**. Rate-change unit equal to one hundredth of one percentage point. *Units:* decimal rate.

s_{MSQ} **CDS market-standard quote**. Conventional spread that is the input or the output of the lesson's simplified converter; it equals the zero-upfront par spread within the converter's

implied flat-hazard model and need not equal the contract's fixed running coupon. *Units:* decimal per year in calculations; basis points per year when explicitly quoted.

N **CDS notional**. Reference currency amount that scales the simplified premium and protection legs; a positive amount, not itself a signed leg cash flow. *Units:* stated currency.

$s^\star$ **CDS par spread**. Contractual spread that makes the two positive leg magnitudes equal at valuation time with zero upfront amount, solved under the lesson's input curves, recovery, timing, and accrued-premium convention. *Units:* decimal per year in calculations; basis points per year when explicitly quoted.

A_0^{prem} **CDS premium annuity**. Present-value coefficient that multiplies contractual spread and notional in the simplified premium leg; positive, and includes scheduled premiums and exact accrued premium under the lesson's default-time model. *Units:* model-years of present value per unit notional.

PV_0^{prem} **CDS premium leg present value**. Positive valuation-time magnitude of the simplified protection buyer's premium payments; its signed contribution to protection-buyer net value is negative. *Units:* stated currency at valuation time.

PV_0^{prot} **CDS protection leg present value**. Positive valuation-time magnitude of the simplified loss-given-default payment received by the protection buyer after a modeled default. *Units:* stated currency at valuation time.

c_{std} **CDS standard coupon**. Fixed annualized rate used to determine the contract's running premium cash flows in the lesson's standard-coupon model; a positive rate paid by the protection buyer on surviving notional and as accrued premium after default. *Units:* decimal per year in calculations; basis points per year when explicitly quoted.

U_0 **CDS upfront amount**. Time-zero cash amount that balances protection and fixed-coupon premium value under the lesson's pricing convention; positive means paid by the protection buyer and negative means received by the protection buyer. *Units:* stated currency at valuation time.

m **Compounding frequency**. Number of equal compounding periods per year under the stated rate convention, a positive integer fixed by the model convention. *Units:* compounding periods per year.

$D(0,t)$ **Discount factor**. Converts one deterministic future unit into value at valuation time. *Units:* current currency-units per future currency-unit.

λ **Constant hazard rate**. Constant conditional default intensity used by the lesson's simplified exponential survival model; a risk-neutral pricing input conditional on survival to the current instant, not a cumulative probability. *Units:* decimal intensity per model-year.

$j^{(m)}$ **Nominal annual rate**. Annualized rate quote that must be paired with its compounding frequency. *Units:* decimal rate per year.

r_m **Periodic rate**. Rate applied once in each compounding period under the stated convention, derived from the stated nominal annual quote in this model. *Units:* decimal per compounding period.

$S(0,t)$ **Survival probability**. Probability, under the explicitly stated model measure, that no modeled default has occurred between valuation time and a stated future time. *Units:* probability between zero and one.

0 **Valuation time**. Common origin from which later model times and present values are measured. *Units:* years from the valuation date.

Answers to the knowledge checks

Knowledge check 7.2.1: CDS market-standard quote and upfront

1. **(a)** MSQ is the conventional quoted spread treated as a zero-upfront par spread for calibrating the flat hazard rate. MSQ is a lesson-local label for the conventional quoted spread. The toy converter treats it as a zero-upfront par spread to infer one flat risk-neutral hazard rate; the contract can have a different fixed running coupon.

2. **(a)** The 100-basis-point standard coupon. The fixed standard coupon determines running premiums. The MSQ calibrates the pricing model, and the upfront amount bridges the resulting protection value and fixed-coupon premium value.

3. 200000 (accepted within ± 0.01). Convert the rates to 0.015 and 0.010. Then U0 = N A (sMSQ - cstd) = 10,000,000 $\times$ 4.0 $\times$ 0.005 = USD 200,000. It is positive, so the protection buyer pays it.

4. -250000 (accepted within ± 0.01). U0 = 5,000,000 $\times$ 2.5 $\times$ (0.03 - 0.05) = -USD 250,000. The high fixed coupon makes the running premium leg exceed protection value, so the protection buyer receives the negative signed upfront.

5. 150 (accepted within ± 0.001). With the calibrated annuity held fixed, sMSQ = cstd + U0/(N A) = 0.01 + 160,000/(8,000,000 $\times$ 4.0) = 0.015, or 150 basis points per year. A full upfront-to-quote conversion cannot hold A fixed while the quote-implied hazard rate changes.

6. 300 (accepted within ± 0.001). The root is lambda = 0.05 per year: A = (1 - exp(-0.05))/0.05 = 0.97541151 and A $\times$ (0.60 $\times$ 0.05 - 0.01) = 0.0195082302, matching U0/N. The quote is 0.60 $\times$ 0.05 = 0.03, or 300 basis points per year.