Chapter 7.4

Equivalent notional and quote risk

From *Credit Products Playground*, with changes after version 0.4.0, not yet released. Interactive version: https://danieltuzes.github.io/credit-and-derivatives/cds/equivalent-notional-and-quote-risk/

7.4.1 What you will be able to do

After this lesson, you should be able to:

- calculate a CDS position's signed value change for a one basis point increase in the liquid tenor's quote, re-solving the hazard rate rather than holding the premium annuity fixed;
- explain why, in a one-factor model, risk measured against the market-standard quote, the upfront, or the par spread is the same risk;
- calculate the equivalent notional and side of the most liquid contract that offsets a position, and its equivalent ratio;
- state what an equivalent-ratio hedge does not offset.

7.4.2 One market level moves everything

The preceding lessons value every contract on a reference entity with one flat Constant hazard rate λ. The quote lesson showed how the market level determines λ: the cds-market-standard-quote s_{MSQ} of the standard contract is treated as a par spread, and λ is the root of (7.2.4). In this lesson, that contract is the most liquid one, with the local cds-liquid-tenor-maturity T_L. In practice, the most liquid tenor is usually five years [1] [2].

A position is one contract on the same reference entity with its own maturity, fixed coupon, and positive local cds-position-notional N_{pos}. Its legs are the exact default-time expectations of the legs lesson, evaluated at the same λ. Let the local cds-position-side σ be $+1$ for bought protection and -1 for sold protection. The local cds-position-value is the protection-buyer value of the legs lesson, multiplied by the side:

$$V^{\mathrm{pos}} = \sigma \left(PV_0^{\mathrm{prot}} - PV_0^{\mathrm{prem}} \right) \tag{7.4.1}$$

Hull marks a CDS to market in the same way, with opposite signs for buyer and seller [3]. Any upfront exchanged at inception is a sunk cash flow and is left out of V^{pos}; it does not change any sensitivity.

For the liquid contract, the local cds-liquid-unit-value is the buyer value per unit of notional:

$$v_L = \frac{PV_0^{\text{prot}}(T_L)}{N} - c_{\text{std}} A_0^{\text{prem}}(T_L) \tag{7.4.2}$$

This value is the fair upfront fraction of the quote lesson. So at the current market level, it equals the upfront amount of the liquid contract per unit of notional [4].

Because λ is the only state variable, every quotation of the liquid contract is a function of λ. Write the local cds-liquid-quotation q for any one of them: the market-standard quote, the upfront fraction, or the par spread. In this model, each quotation is strictly increasing in λ, so each quotation is invertible. So a change of the quote is a change of the hazard rate, whichever quotation the trading desk uses for its marks. The model has one risk factor, expressed in three quotations. In this model, with a flat hazard rate and a flat interest rate, the par spread is also the same at every maturity. So the par spread of a position and the liquid quote are the same number.

7.4.3 Risky DV01 by bump and reprice

Tuckman and Serrat define the risky DV01 of a CDS as the change in its value for a one basis point change in the CDS spread, computed by finding the hazard rate that moves the spread by one basis point and revaluing the contract [5]. This lesson uses that bump-and-reprice definition with the local cds-quote-bump $h = 0.0001$ per year, one basis-point of market-standard quote.

Write $\lambda(s_{\text{MSQ}})$ for the hazard rate re-solved from a quote. The local cds-position-quote-sensitivity is the change in the position value when the quote increases by h:

$$\text{DV01}^{\text{pos}} = V^{\text{pos}}\Big(\lambda\left(s_{\text{MSQ}} + h\right)\Big) - V^{\text{pos}}\Big(\lambda\left(s_{\text{MSQ}}\right)\Big) \tag{7.4.3}$$

The local cds-liquid-unit-quote-sensitivity is the same difference, for one unit of bought liquid protection:

$$\text{DV01}^{L} = v_L\Big(\lambda\left(s_{\text{MSQ}} + h\right)\Big) - v_L\Big(\lambda\left(s_{\text{MSQ}}\right)\Big) \tag{7.4.4}$$

Three properties of the definition are important in practice.

- **Both legs change.** A higher hazard rate increases the protection leg and decreases the premium annuity. So the sensitivity is not the annuity multiplied by one basis point. Holding the annuity fixed is the shortcut that the quote lesson warned against.
- **The sign depends on the side.** Bought protection gains value when the quote increases, so its sensitivity is positive. Sold protection has a negative sensitivity. DV01^{L} is defined for bought protection and is positive.
- **The bump is finite.** The value is a nonlinear function of the quote. So the result of a one basis point bump differs slightly from the derivative. The section “What an equivalent-ratio hedge does not do” discusses this difference.

7.4.4 The liquid-tenor equivalent

A hedge that removes the quote risk is a position in the liquid contract whose notional makes the sensitivity of the hedged position zero. Let the local cds-equivalent-notional N_{eq} be the notional of bought liquid protection. The hedge condition and its solution are:

$$\mathrm{DV01}^{\mathrm{pos}} + N_{\mathrm{eq}}\,\mathrm{DV01}^{L} = 0 \qquad \Longleftrightarrow \qquad N_{\mathrm{eq}} = -\frac{\mathrm{DV01}^{\mathrm{pos}}}{\mathrm{DV01}^{L}} \tag{7.4.5}$$

A positive N_{eq} means: buy protection in the liquid tenor. A negative N_{eq} means: sell protection in the liquid tenor. Because $\mathrm{DV01}^{L}$ is positive, the sign of N_{eq} is always opposite to the sign of the position's sensitivity. The local cds-equivalent-ratio is the size of the hedge relative to the position:

$$\rho_{\mathrm{eq}} = \frac{|N_{\mathrm{eq}}|}{N_{\mathrm{pos}}} \tag{7.4.6}$$

This hedge is the DV01-neutral hedge that Tuckman and Serrat describe for bonds. There, the face amount of a liquid hedge is minus the ratio of the two DV01s, multiplied by the position amount. The instrument with the larger sensitivity is traded in the smaller quantity [6]. The names **equivalent notional** and **equivalent ratio**, for the hedge amount and its ratio, belong to this course (`NEEDS_SOURCE` for desk usage; the registered sources define the DV01 hedge but not these names).

The choice of quotation does not change the hedge. A trading desk can measure the position's risk against the market-standard quote, the upfront, or a par spread. In this model, the choice does not change the equivalent notional. In the limit of a small bump, both sensitivities are derivatives with respect to the same quotation q. The chain rule through the one state variable λ gives:

$$N_{\mathrm{eq}} = -\frac{dV^{\mathrm{pos}}/dq}{dv_L/dq} = -\frac{(dV^{\mathrm{pos}}/d\lambda)\,(d\lambda/dq)}{(dv_L/d\lambda)\,(d\lambda/dq)} = -\frac{dV^{\mathrm{pos}}/d\lambda}{dv_L/d\lambda} \tag{7.4.7}$$

The factor $d\lambda/dq$, which converts a change of the hazard rate into a change of the quotation, cancels. So MSQ risk, upfront risk, and par spread risk are one risk, and the same equivalent notional offsets all three. The sensitivities themselves differ across quotations, because each sensitivity is measured per unit of a different quotation. The hedge amount is the same. With a finite bump, the three equivalent ratios differ only because of the curvature, as the worked example shows.

Example 7.4.1: Equivalent-notional hedges

A seven-year position hedged with five-year

The liquid contract is a five-year standard contract with quarterly premiums, $c_{\mathrm{std}} = 0.010$ per year (100 basis points per year) and market-standard quote $s_{\mathrm{MSQ}} = 0.016$ per year (160 basis points per year). The risk-free rate is 2% per year, continuously compounded, and the recovery rate is 40%. Calibration to the quote gives $\lambda = 0.026600129$ per year. By (7.4.2), one unit of bought five-year protection is worth $v_L = 0.026694561$ before the upfront amount.

The position is bought protection on a seven-year contract with the same coupon of 100 basis points and $N_{\mathrm{pos}} =$ USD 10,000,000. At the calibrated hazard rate, the protection-leg present value of the position is USD 953,285.99, and its premium-leg present value is USD 595,803.74. So by (7.4.1), $V^{\mathrm{pos}} =$ USD 357,482.24.

Bump the liquid quote by $h = 0.0001$, to 161 basis points. The re-solved hazard rate is 0.026766381 per year. Revaluing with (7.4.3) and (7.4.4) gives the two sensitivities:

$$\mathrm{DV01}^{\mathrm{pos}} = 363{,}240.36 - 357{,}482.24 = \text{USD } 5{,}758.12 \text{ per basis point}$$

$$\mathrm{DV01}^{L} = 0.000434070474 \text{ per unit notional per basis point}$$

By (7.4.5), the equivalent notional is:

$$N_{\text{eq}} = -\frac{5{,}758.117764}{0.000434070474} = -\text{USD } 13{,}265{,}398.37$$

So the hedge sells protection on a notional of USD 13,265,398 of the five-year contract. By (7.4.6), $\rho_{\text{eq}} = 1.3265$. The seven-year contract has the larger sensitivity per unit of notional. So the hedge notional in the five-year contract is larger than the notional of the position. The tested implementation reports a hedged sensitivity of zero to arithmetic precision.

A sold three-year position at a 500 basis point coupon

Keep the same liquid contract and market level. The position is sold protection on a three-year contract with coupon 500 basis points per year and $N_{\text{pos}} = \text{USD } 20{,}000{,}000$, so $\sigma = -1$. Because the coupon of 500 basis points is higher than the market level of 160 basis points, the seller's value is positive: $V^{\text{pos}} = \text{USD } 1{,}899{,}072.40$.
After the same bump of one basis point, the seller's value decreases to USD 1,893,025.81. So the sensitivity is:

$$\text{DV01}^{\text{pos}} = -\text{USD } 6{,}046.59 \text{ per basis point}$$

The seller loses value when the quote increases. With the same $\text{DV01}^{L} = 0.000434070474$, the equivalent notional is:

$$N_{\text{eq}} = -\frac{-6{,}046.591823}{0.000434070474} = +\text{USD } 13{,}929{,}977.23$$

So the hedge buys protection on a notional of USD 13,929,977 of the five-year contract, and $\rho_{\text{eq}} = 0.6965$. The three-year contract has a smaller sensitivity per unit of notional than the five-year contract. So a five-year notional smaller than the position notional offsets the position. This holds although the high coupon gives the position a large value.

Three quotations, one hedge

Repeat the seven-year hedge, but bump the upfront fraction of the liquid contract by 0.0001 (one basis point of notional) instead of its quote. Re-solve the hazard rate from the bumped upfront fraction. The equivalent ratio is 1.32670, compared with 1.32654 from the quote bump. With both bumps one thousand times smaller, both ratios are 1.32674, as (7.4.7) predicts. So the difference at one basis point is a result of the curvature of a finite bump, not of a different risk.
As a check, apply the construction to the liquid contract itself, with bought protection on USD 10,000,000. The equivalent ratio is exactly one, on the opposite side: the hedge is the offsetting trade.

7.4.5 What an equivalent-ratio hedge does not do

The hedge sets one sensitivity to zero, at one market level, in one model. It leaves at least the following exposures, and a trading desk manages each of them separately.

- **Default.** The seven-year hedge above sells protection on USD 13,265,398 against bought protection on USD 10,000,000. If default occurs immediately, with 40% recovery, the position receives $\text{LGD} \times 10{,}000{,}000 = \text{USD } 6{,}000{,}000$ and the hedge pays $0.6 \times 13{,}265{,}398.37 = \text{USD } 7{,}959{,}239.02$. The net loss is about USD 1,959,239, before accrued premium. Equal

quote sensitivities do not imply equal default payoffs. So the jump-to-default exposure of a hedged portfolio must be measured separately.

- **Curve shape.** The model has one hazard rate. Under the fitted curve of the credit-curve lesson, a seven-year position also depends on the segments beyond five years, on which the five-year contract does not depend. A change of the five-year quote alone would change the value of only part of the position. Measuring the sensitivity separately for each tenor is the standard extension (`NEEDS_SOURCE`: the registered sources describe single-factor DV01 hedging only).
- **Curvature.** Both sensitivities change when the quote changes. So the hedge stops being exact and must be rebalanced. Tuckman and Serrat make the same point for bond DV01 hedges [7].
- **Inputs held fixed.** The recovery rate, the discount curve, and the accrual and settlement conventions are constant in the model. A change in any of them changes the values of the two contracts by different amounts.
- **Costs.** The hedge pays the bid-offer spread of the liquid contract. The lower liquidity of the position's own tenor is the reason why the hedge is not the offsetting trade in that tenor [8].

7.4.6 Check your understanding

The assessment separates the bump-and-reprice sensitivity, including its sign for a seller, from sizing the equivalent notional and ratio. Each competency has direct and transfer evidence.

Knowledge check 7.4.1: Equivalent notional and quote risk

1. A bought-protection position is worth USD 357,482.24 at the liquid five-year quote of 160 basis points and USD 363,240.36 after the quote is bumped to 161 basis points with the flat hazard rate re-solved. Calculate the position's risky DV01 in USD per basis point.
2. A trader has sold protection on a three-year contract. Valued for a protection buyer, the contract is worth USD -1,899,072.40 at the liquid quote and USD -1,893,025.81 after a one basis point bump with the hazard rate re-solved. Calculate the seller's risky DV01 in USD per basis point, using the sign convention that a positive number is a gain for the seller.
3. A bought-protection position has risky DV01 of USD 5,758.117764 per basis point. One unit of notional of bought protection in the liquid five-year contract has risky DV01 of 0.000434070474 per basis point. Calculate the magnitude in USD of the liquid-tenor equivalent notional that offsets the position.
4. A sold-protection position with notional USD 20,000,000 has risky DV01 of USD -6,046.591823 per basis point for the seller. One unit of notional of bought protection in the liquid five-year contract has risky DV01 of 0.000434070474 per basis point. Calculate the equivalent ratio of the liquid-tenor hedge to the position notional.

Answers: end of the chapter.

7.4.7 Model boundary and review note

This AI-assisted lesson and all cited locators remain `draft` pending human editorial and quantitative review. All input values are invented for teaching. The sensitivity is a one basis point bump of the liquid tenor's market-standard quote, under one flat hazard rate, one deterministic discount rate, one deterministic recovery rate, and a regular quarterly model-year schedule. The hedge is sized at one market level and is not rebalanced. The lesson omits curve shape risk, default and recovery risk beyond the illustration above, bid-offer, transaction costs, calendar and settlement conventions, counterparty risk, collateral, funding, and any stochastic model of rates or intensity.

References

[1] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.1, printed p. 549.

[2] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.5, printed p. 365.

[3] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.2, printed pp. 553-554.

[4] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.4, printed p. 557.

[5] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.10, printed p. 379.

[6] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 4 §4.3, printed pp. 110-111, eqs. 4.6-4.9.

[7] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 4 §4.2, printed p. 109 (DV01 as a local measure).

[8] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.6, printed pp. 366-367 (unwinding through the on-the-run contract and managing the maturity and spread mismatch).

Notation

N_{eq} **CDS equivalent notional**. Signed notional of the liquid contract, bought as protection, whose quote sensitivity cancels the position sensitivity; a negative value means protection is sold in the liquid tenor. *Units:* stated currency.

ρ_{eq} **CDS equivalent ratio**. Absolute equivalent notional divided by position notional: how much of the most liquid contract offsets one unit of the position quote risk. It is positive; the side is stated separately.

$$\rho_{\text{eq}} = |N_{\text{eq}}|/N_{\text{pos}}$$

q **CDS liquid quotation**. Any one quotation of the liquid contract viewed as a function of the flat hazard rate: its market-standard quote, its upfront fraction, or its par spread. Each is a strictly increasing function of the hazard rate in this model.

T_L **CDS liquid tenor maturity**. Maturity of the most liquid standard contract on the reference entity, whose market-standard quote gives the single market level of the flat-hazard model. *Units:* model-years from valuation time.

$\mathrm{DV01}^L$ **CDS liquid unit quote sensitivity**. Change in the value of one unit of notional of bought liquid-tenor protection when the liquid quote rises by the bump. It is positive, because bought protection gains when the quote rises. *Units:* fraction of notional per basis point.

v_L **CDS liquid unit value**. Value at valuation time of one currency unit of notional of bought protection in the liquid contract, before upfront. It is a fraction of notional. *Units:* fraction of notional at valuation time.

N_{pos} **CDS position notional**. Positive notional of the position being hedged, in currency units. It carries no sign; the position side does. *Units:* stated currency.

$\mathrm{DV01}^{\mathrm{pos}}$ **CDS position quote sensitivity**. Change in the signed position value when the liquid tenor market-standard quote rises by the bump, with every contract revalued at the re-solved flat hazard rate. Market usage calls this a risky DV01 or spread DV01. *Units:* stated currency per basis point.

σ **CDS position side**. Sign of the position side: plus one for bought protection and minus one for sold protection. It converts protection-buyer value into value for the position holder.

$$\sigma \in \{+1, -1\}$$

V^{pos} **CDS position value**. Signed value of the position for its own side at valuation time, before any upfront exchanged at inception: the protection-buyer value of the contract multiplied by the side sign. *Units:* stated currency at valuation time.

h **CDS quote bump**. Size of the bump applied to the liquid tenor quote in the bump-and-reprice sensitivity. The lesson uses one basis point of market-standard quote. *Units:* decimal rate per year.

$A(0,t)$ **Accumulation factor**. Grows one current unit over a stated future horizon under the selected rate model. *Units:* dimensionless currency-units per current currency-unit.

$1\,\mathrm{bp}$ **Basis point**. Rate-change unit equal to one hundredth of one percentage point. *Units:* decimal rate.

s_{MSQ} **CDS market-standard quote**. Conventional spread that is the input or the output of the lesson's simplified converter; it equals the zero-upfront par spread within the converter's implied flat-hazard model and need not equal the contract's fixed running coupon. *Units:* decimal per year in calculations; basis points per year when explicitly quoted.

N **CDS notional**. Reference currency amount that scales the simplified premium and protection legs; a positive amount, not itself a signed leg cash flow. *Units:* stated currency.

A_0^{prem} **CDS premium annuity**. Present-value coefficient that multiplies contractual spread and notional in the simplified premium leg; positive, and includes scheduled premiums and exact accrued premium under the lesson's default-time model. *Units:* model-years of present value per unit notional.

PV_0^{prem} **CDS premium leg present value**. Positive valuation-time magnitude of the simplified protection buyer's premium payments; its signed contribution to protection-buyer net value is negative. *Units:* stated currency at valuation time.

PV_0^{prot} **CDS protection leg present value**. Positive valuation-time magnitude of the simplified loss-given-default payment received by the protection buyer after a modeled default. *Units:* stated currency at valuation time.

c_{std} **CDS standard coupon**. Fixed annualized rate used to determine the contract's running premium cash flows in the lesson's standard-coupon model; a positive rate paid by the protection buyer on surviving notional and as accrued premium after default. *Units:* decimal per year in calculations; basis points per year when explicitly quoted.

m **Compounding frequency**. Number of equal compounding periods per year under the stated rate convention, a positive integer fixed by the model convention. *Units:* compounding periods per year.

$D(0,t)$ **Discount factor**. Converts one deterministic future unit into value at valuation time. *Units:* current currency-units per future currency-unit.

λ **Constant hazard rate**. Constant conditional default intensity used by the lesson's simplified exponential survival model; a risk-neutral pricing input conditional on survival to the current instant, not a cumulative probability. *Units:* decimal intensity per model-year.

LGD **Loss given default**. Fraction of an explicitly stated reference amount not recovered under a deterministic recovery convention, relative to the same reference amount used by the recovery rate. *Units:* decimal fraction between zero and one.

$j^{(m)}$ **Nominal annual rate**. Annualized rate quote that must be paired with its compounding frequency. *Units:* decimal rate per year.

r_m **Periodic rate**. Rate applied once in each compounding period under the stated convention, derived from the stated nominal annual quote in this model. *Units:* decimal per compounding period.

R **Recovery rate**. Fraction of a stated reference amount recovered after a modeled default under an explicitly stated recovery convention; a non-negative fraction whose reference amount, payment timing, and settlement convention are set by the model that uses it. *Units:* decimal fraction between zero and one.

$S(0,t)$ **Survival probability**. Probability, under the explicitly stated model measure, that no modeled default has occurred between valuation time and a stated future time. *Units:* probability between zero and one.

0 **Valuation time**. Common origin from which later model times and present values are measured. *Units:* years from the valuation date.

Answers to the knowledge checks

Knowledge check 7.4.1: Equivalent notional and quote risk

1. 5758.12 (accepted within ± 0.01). The bump-and-reprice sensitivity is the value after the bump minus the value before it: 363,240.36 - 357,482.24 = USD 5,758.12 per basis point. It is positive because bought protection gains when the quote rises.

2. -6046.59 (accepted within ± 0.01). The seller's value is minus the buyer's value: USD 1,899,072.40 before and USD 1,893,025.81 after the bump. The seller's sensitivity is 1,893,025.81 - 1,899,072.40 = USD -6,046.59 per basis point; a seller loses when the quote rises.

3. 13265398.4 (accepted within ± 1). $N_{\text{eq}} = \frac{-5{,}758.117764}{0.000434070474} = -$USD 13,265,398.4. The negative sign means the hedge sells protection in the five-year contract; its magnitude is USD 13,265,398.4.

4. 0.6965 (accepted within ± 0.0001). $N_{\text{eq}} = \frac{-(-6{,}046.591823)}{0.000434070474} = +$USD 13,929,977.2, so the hedge buys protection in the five-year contract. The equivalent ratio is 13,929,977.2 / 20,000,000 = 0.6965.