# Chapter 7.3

# Credit curve and market observables

From *Credit Products Playground*, with changes after version 0.4.0, not yet released. Interactive version: https://danieltuzes.github.io/credit-and-derivatives/cds/credit-curve-and-market-observables/

## 7.3.1 What you will be able to do

After this lesson, you should be able to:

- classify each quantity in a standard CDS valuation as market-observed, contractual, conventional, or model-solved;
- explain why the traded upfront alone is enough to determine the pricing hazard rate, and why that is a consequence of risk-neutral pricing rather than an extra assumption;
- read a credit curve as tenor marks together with the survival curve fitted to them, and tell a change of quotation from a curve fit;
- calculate the survival probability implied by fitted hazard segments and check that a tenor reprices to its mark.

## 7.3.2 A traded price is an expectation

The risk-neutral lesson established that, in an arbitrage-free model with the cash account as numeraire, a claim's present-value is the expectation under the risk-neutral-probability-measure $\mathbb{Q}$ of its discounted payoff; see (1.7.2). Shreve states the general discounted-payoff formula [1].

A CDS is such a claim. Let the local cds-buyer-discounted-cash-flows $\Pi^{\text{buyer}}$ be the random sum of the protection buyer's discounted signed cash flows over the life of the contract, before any upfront amount. It is the protection received at the modeled default time minus the scheduled and accrued premium paid. Each cash flow is discounted from its own payment time to valuation-time 0. The present value of the CDS is the risk-neutral expectation of this sum:

$$PV_0 = \mathbb{E}^{\mathbb{Q}}\left[\Pi^{\text{buyer}}\right] \tag{7.3.1}$$

Now add the one fact that the market gives. A new standard contract is traded at its market upfront amount: the buyer pays the cds-upfront-amount $U_0$ at time zero and receives the claim valued by (7.3.1). A trade at the market level has zero present value at inception. So the upfront amount equals the present value:

$$U_0 = \mathbb{E}^{\mathbb{Q}}\left[\Pi^{\text{buyer}}\right] = PV_0^{\text{prot}} - PV_0^{\text{prem}} \tag{7.3.2}$$

The second equality follows from the linearity of expectation: the expectations of the protection cash flows and of the premium cash flows are calculated separately. They are the two positive leg present values of the preceding lessons. Equation (7.3.2) is the same identity as (7.2.2), read in the other direction. In the preceding lesson, the model parameters were inputs and the upfront was the output. Here, the upfront is observed, and the equation constrains the model.

**Why one equality is enough.** The standard contract has no cash flow at inception other than the upfront amount. So the trade gives exactly one number. In the model of this course, discounting and recovery are deterministic, and both legs are linear expectations over the default time. So the only unknown in (7.3.2) is the timing of default under the pricing measure: the survival-probability curve. Setting the expected values of the two legs equal, with the upfront amount on the premium side, is therefore not a modeling choice added to risk-neutral pricing. It is what risk-neutral pricing says about a contract that has just been traded at the market level. Tuckman and Serrat describe the market convention in these steps. First, find the hazard rate that makes the expected discounted fee leg and contingent leg (in this course, the premium leg and the protection leg) equal. Then value the standard coupon at that hazard rate [2].

One number determines one unknown. With a single flat Constant hazard rate $\lambda$ per contract, as in the preceding lesson, one traded price is exactly enough. With a term structure of hazard rates, one traded price per tenor is exactly enough only if the curve has one free parameter per tenor. The piecewise-constant curve below has this property. With fewer prices than parameters, the curve is under-determined, and a convention must fix the remaining parameters. With more prices than parameters, the curve can in general only fit the prices approximately (a best fit).

### 7.3.3 What the market shows, the model assumes, and the model solves

If every quantity is called an input, it is not clear which quantities the market can contradict. Table 7.3.1 classifies the quantities of a standard contract at one tenor.

The risk-neutral interpretation has two consequences.

- **The solved probabilities are pricing weights.** Hull describes implying default probabilities from CDS quotes as the analogue of implying volatilities from option prices. Hull states that CDS valuation needs risk-neutral, not real-world, default probabilities [10]. A survival curve fitted to quotes is a statement about prices, not a forecast of default frequencies. Tuckman and Serrat make the same point for the price-implied hazard rate [11].
- **Recovery must be assumed because one price cannot identify two unknowns.** The protection payoff is proportional to LGD, and the implied hazard rate is approximately proportional to 1/LGD. So a contract valued with the same recovery rate that implied its hazard rate is not very sensitive to the choice of recovery rate. A binary CDS, whose payoff does not depend on recovery, would add the second equation needed to separate the two [12].

### 7.3.4 The credit curve is two objects

A discount curve is a set of dated discount factors in one currency and on one funding basis; it is not one yield (see the rates lesson). A credit curve is the same idea for one reference entity. But a

Table 7.3.1 **Observed, contractual, convention, and solved quantities.** Each quantity of a standard contract at one tenor: whether the market shows it, the contract fixes it, a convention assumes it, or the model solves it, and where it comes from.

| Quantity | Status | Where it comes from |
|---|---|---|
| Signed upfront $U_0$, or the cds-market-standard-quote it stands for | **Observed** | Dealer bid and offer for the standard contract; a mid is taken as the mark. [3] |
| Risk-free discount-factor $D(0,t)$ | **Observed** in another market, then an input | Built from the rates market, not from the CDS; the converter takes it as a discount input. [4] |
| cds-standard-coupon $c_{\text{std}}$, cds-notional $N$, maturity, credit events, reference obligations | **Contractual** | Written into the standard contract; the coupon is 100 or 500 basis points and maturities fall on standard dates. [5] [6] |
| recovery-rate $R$ and hence loss-given-default LGD | **Convention** | Assumed by the converter, commonly 40% but 20% or 25% for some contract types. It is not observed in the quote. [7] |
| Shape of the hazard rate between and beyond tenors, accrual treatment | **Convention** | The converter's single flat hazard rate, or this lesson's piecewise-constant curve. [8] |
| Constant hazard rate $\lambda$, the survival curve, the cds-par-spread $s^\star$, the other quotation | **Solved** | Root of (7.3.2) given everything above. [9] |

credit curve has two layers, and this course keeps them separate.

**The marked curve.** Use the local cds-tenor-index $k$ across the cds-tenor-count $K$ quoted tenors with maturities $T_1 < \cdots < T_K$. At each tenor, the market shows the price of the standard contract. A mark is the recorded price at one tenor. If the trading desk records the price as a conventional spread, the mark is the local cds-tenor-quote $s_k$. If the desk records the traded price itself, the mark is the local cds-tenor-upfront-fraction $u_k$. This course calls a curve whose marks are market-standard quotes an **MSQ-marked curve**, and one whose marks are upfront fractions an **upfront-marked curve**. Both labels are local to this course (`NEEDS_SOURCE`: the desk vocabulary for marking conventions is not documented in the registered sources). Tuckman and Serrat tabulate five-year spreads for several reference entities. They also plot the term structure of spreads across tenors: its slope is normally upward, but it is inverted in periods of stress [13]. Hull lists the tenors that commonly trade; the five-year tenor is the most liquid [14].

**The fitted curve.** The legs lesson allows a deterministic time-varying hazard rate. It shows that the survival probability is the exponential of minus the integral of the hazard rate. The fitted credit curve is the special case in which the hazard rate is constant on each segment between consecutive tenors: the local cds-segment-hazard-rate $\lambda_k$ on $(T_{k-1}, T_k]$, with $T_0 = 0$. The survival probability is then a product with one factor per segment:

$$S^{\mathbb{Q}}(0,T_k) = S^{\mathbb{Q}}(0,T_{k-1}) \exp\Big(-\lambda_k \left(T_k - T_{k-1}\right)\Big), \qquad S^{\mathbb{Q}}(0,0) = 1 \tag{7.3.3}$$

Inside a segment, the survival probability decreases at the constant hazard rate of that segment, as in the constant-hazard lesson. Hull notes that when spreads for different maturities are available, the hazard rate is calculated as a step function [15].

The marked curve is data. The fitted curve is a model object, chosen so that the model reprices the data. The next distinction depends on keeping the two layers separate.

### 7.3.5 Transforming marks is not fitting the curve

**Curve transformation** changes the quotation of each mark and does not build a term structure. For each tenor separately, the standard converter takes $s_k$ and solves for a flat hazard rate for that tenor alone. This course writes this rate as the local cds-flat-tenor-hazard-rate $\lambda_k^{\text{flat}}$. The converter then values the fixed coupon at that rate and returns $u_k$. The converter can also do the same steps in the reverse direction. This is the conversion of the preceding lesson, applied once per tenor [16]. The flat hazard rates of different tenors are solutions of separate equations. They are not one survival curve, and the official converter itself distinguishes its single flat hazard rate from a term structure [17]. The same tenor-by-tenor conversion also gives a third quotation: the par spread of a hypothetical zero-upfront contract. The name **curve transformation** belongs to this course (`NEEDS_SOURCE` for the phrase itself).

**Fitting the curve** finds one term structure that reprices every mark at once. With the marks written as upfront fractions, the fit condition at tenor $k$ is the buyer-value identity (7.3.2), evaluated with the segment hazard rates up to $k$:

$$PV_0^{\text{prot}}(T_k; \lambda_1, \dots, \lambda_k) - c_{\text{std}} N A_0^{\text{prem}}(T_k; \lambda_1, \dots, \lambda_k) = u_k N \tag{7.3.4}$$

Both legs are the exact default-time integrals of the legs lesson, with the piecewise-constant hazard rate substituted. Each premium period lies inside one segment. So each period is a one-period flat-hazard model that starts from the survival probability and the discount factor at the start of the period. Solve (7.3.4) in maturity order. The first tenor determines $\lambda_1$. Then $\lambda_1$ is held fixed, and the second tenor determines $\lambda_2$, and so on. Each step is a one-dimensional root solve for $\lambda_k$. The buyer value of a contract increases with the hazard rate of any segment inside the life of the contract, so each step has at most one root. A zero-upfront par mark is the special case $u_k = 0$, with the coupon set to the quoted par spread. The result is a curve under which every quoted tenor reprices exactly. The survival probabilities of this curve at all tenors are consistent with each other.

The same survival curve then prices every other contract on the same reference entity: a maturity between the quoted tenors, an existing contract with a different coupon, or a hypothetical zero-upfront contract. This is the purpose of a fitted curve. The fit has two limits. First, the fitted par spread at a tenor is in general not equal to the MSQ mark of that tenor. The reason is that the fitted premium annuity differs from the flat annuity of the converter, although both reprice the same traded upfront amount. This is the "par spread from another curve" point of the preceding lesson. Second, a solution need not exist. A later mark that is cheaper than the zero-hazard value implied by the earlier segments would need a negative $\lambda_k$. The model rejects a negative hazard rate, because a survival probability cannot increase with time.

**Example 7.3.1: A credit curve from market marks**

**Transform the marks**

Take an MSQ-marked curve for one reference entity: standard contracts with quarterly premiums and a fixed coupon $c_{\text{std}} = 0.010$ per year (100 basis points per year), continuously compounded risk-free rate 2% per year, recovery $R = 0.40$, and marks $s_1 = 0.008$, $s_2 = 0.012$, $s_3 = 0.016$ at maturities $T_1 = 1$, $T_2 = 3$, $T_3 = 5$ model-years. Notional is USD 10,000,000.
For each tenor separately, the flat converter of the preceding lesson gives:

| Tenor $k$ | $s_k$ (bp/yr) | $\lambda_k^{\text{flat}}$, per yr | Flat $A_0^{\text{prem}}$, model-yrs | $u_k$ | $U_0$, USD | Flat $S(0,T_k)$ |
|---|---|---|---|---|---|---|
| 1 | 80 | 0.013300046 | 0.981077842 | −0.001962156 | −19,621.56 | 0.986788009 |
| 2 | 120 | 0.019950083 | 2.820140305 | 0.005640281 | 56,402.81 | 0.941905574 |
| 3 | 160 | 0.026600129 | 4.449093452 | 0.026694561 | 266,945.61 | 0.875464525 |

Each row is the conversion of the preceding lesson. For example, at five years, $u_3 = 4.449093452 \times (0.016 - 0.010) = 0.026694561$. The one-year mark is below the coupon, so the buyer receives the upfront amount. The three flat hazard rates are solutions of three separate equations. The last column is not one survival curve: for example, it does not define the survival probability at two years.

**Fit the curve**

Now fit a piecewise-constant hazard curve to the three upfront fractions in maturity order.

| Tenor $k$ | Segment, model-yrs | $\lambda_k$, per yr | Fitted $S(0,T_k)$ | Fitted $A_0^{\text{prem}}$, model-yrs | Fitted $PV_0^{\text{prot}}/N$ | Fitted $s^\star$ (bp/yr) |
|---|---|---|---|---|---|---|
| 1 | (0, 1] | 0.013300046 | 0.986788009 | 0.981077842 | 0.007848623 | 80.0000 |
| 2 | (1, 3] | 0.023463765 | 0.941550234 | 2.829254583 | 0.033932826 | 119.9356 |
| 3 | (3, 5] | 0.037572831 | 0.873389867 | 4.500220799 | 0.071696769 | 159.3183 |

Every tenor reprices. For example, at five years, $0.071696769 - 0.010 \times 4.500220799 = 0.026694561 = u_3$. In the tested implementation, the residuals at all three tenors are below $10^{-12}$.

Compare this table with the transformation. The hazard rate of the first segment equals the one-year flat rate, because a one-year contract depends only on the first segment. The hazard rates of the later segments are higher than the flat rates. The five-year flat rate, 0.026600, is an average over five years. The fitted hazard rate on the last two years is 0.037573, because the shorter tenors, with lower hazard rates, already price the earlier years. So the fitted five-year survival probability, 0.873390, is below the flat survival probability, 0.875465. The fitted par spreads at three and five years, 119.94 and 159.32 basis points, are not equal to the marks of 120 and 160 basis points. The reason is that the fitted curve reproduces the same traded upfront amounts with different premium annuities.

The fitted curve also defines the survival probability between tenors. By (7.3.3), the survival probability at two years is $0.986788009 \exp(-0.023463765 \times 1) = 0.963903771$, and at four years it is $0.941550234 \exp(-0.037572831 \times 1) = 0.906829881$.

**The marks are prices of particular contracts**

Now read the same three numbers, 80, 120, and 160 basis points, as zero-upfront par spreads, not as MSQ marks with a coupon of 100 basis points. So $u_k = 0$, and the coupon of each tenor equals its quote. The fit then gives segment hazard rates of 0.013300046, 0.023480220, and 0.037863368 per year, and a five-year survival probability of 0.872853786 instead of 0.873389867. The first segment is unchanged, because the converter also treats the one-year MSQ of 80 basis points as the par spread of a zero-upfront one-year contract. The later segments change. A standard contract with a coupon of 100 basis points and an upfront amount is a different stream of cash flows from a zero-upfront contract with a coupon of 120 basis points. The two prices are written with the same number, but they are prices of different contracts. A mark is the price of a particular contract, and the fit must know which contract.

**When no curve fits**

Suppose the one-year zero-upfront par mark is 300 basis points and the three-year mark is 100 basis points. The first segment prices the one-year contract. Then the three-year contract at

100 basis points is worth more to the seller than the first year alone implies, even with zero default risk after year one. So its mark is below the zero-hazard bound, and the fit reports that the mark would need a negative segment hazard rate. In the tested implementation, a three-year mark of 110 basis points has a solution, with a hazard rate of 0.001573 per year in the second segment.
Steeply inverted marks are not necessarily an arbitrage, but they indicate a problem. Either the marks are out of date or not at market levels, or the piecewise-constant shape and the deterministic recovery of the model cannot describe them.

### 7.3.6 A note on jargon

The vocabulary of CDS trading desks mixes contract terms, quotation conventions, and risk measures. Table 7.3.2 gives this course's reading of the most common phrases, with the registered source where one supports it. Entries marked `NEEDS_SOURCE` are recorded as usage this course adopts locally; a human should confirm or replace them before review.

### 7.3.7 Check your understanding

The assessment separates the observed-versus-solved classification, the transformation-versus-fit distinction, and the survival and repricing arithmetic of a fitted curve. Each competency has direct and transfer evidence.

**Knowledge check 7.3.1: Credit curve and market observables**

1. In the standard conversion of a five-year contract's quote into an upfront, which quantity is solved from the traded price rather than observed in the market or assumed by convention?

   (a) The flat risk-neutral hazard rate

   (b) The fixed running coupon

   (c) The recovery rate

2. A dealer shows five-year protection at 160 basis points bid and 165 basis points offer, and the converter uses a 40% recovery rate. Which statement is correct?

   (a) The 40% recovery is a convention assumed by the converter; the quote does not observe it

   (b) The quote reveals that the market expects 40% recovery

   (c) The dealer is quoting the hazard rate directly

3. An MSQ-marked curve lists 80, 120, and 160 basis points at one, three, and five years. Converting each mark to an upfront with the standard converter, one tenor at a time, is best described as which of the following?

   (a) A change of quotation using a separate flat hazard rate per tenor; it does not build one term structure

   (b) A fit of one piecewise-constant hazard curve that reprices all three tenors together

   (c) A direct observation of the survival probabilities at one, three, and five years

Table 7.3.2 **CDS desk jargon.** Common phrases of CDS trading desks with this course's reading of each and the registered source that supports it. Entries marked NEEDS_SOURCE are local usage for a human to confirm.

| Phrase | Meaning in this course |
|---|---|
| Reference entity, credit event, notional | The named borrower, the contractually defined trigger, and the amount protected. [18] [19] |
| Buying protection, selling protection; short credit, long credit | A protection buyer pays premium and gains on default, like a short seller of the bond; a protection seller resembles a bond holder. [20] |
| Index buyer and seller | For index CDS the words are reversed: the index buyer receives premium and pays on default, like a portfolio holder. [21] |
| Spread, quoted spread, conventional spread, MSQ | The annualized premium of a hypothetical zero-upfront contract, used as the conventional quote; this course's local label is market-standard quote. [22] [14] |
| Par spread | The running spread that makes the two legs equal with zero upfront under a stated curve; see the cds-par-spread card. |
| Standard coupon, points upfront, price | The fixed 100 or 500 basis point running coupon and the upfront that balances it; the upfront per 100 notional is quoted as points, or as a price of 100 minus points. [23] [24] |
| Risky annuity, risky duration, RPV01 | The premium annuity: the present value of one unit of running spread per unit notional, survival-weighted; Hull calls it the duration of the CDS payments. The label RPV01 is `NEEDS_SOURCE`. [24] |
| Risky DV01, spread DV01, CS01 | The change in a CDS value for a one basis point change in the CDS spread, found by re-solving the hazard rate and revaluing; the next lesson uses it. The label CS01 is `NEEDS_SOURCE`. [25] |
| Duration times spread, DTS | A spread-risk measure for bonds that scales spread duration by the spread level, motivated by spreads moving proportionally. [25] |
| On-the-run, off-the-run | The most liquid current standard contract of a tenor versus earlier contracts with older maturity dates. [26] |
| IMM dates, roll | Standard maturity dates on the twentieth of March, June, September, and December; contracts and index series move to new dates on a schedule. [23] [27] |
| Recovery convention | The assumed recovery in the converter: commonly 40%, with 20% or 25% for some contract types. [7] |
| Credit curve, spread curve, term structure of spreads | Marks across tenors and the survival curve fitted to them; upward-sloping in calm markets and inverted in stress. [21] |
| MSQ-marked curve, upfront-marked curve, curve transformation, curve fitting | Local labels defined in this lesson (`NEEDS_SOURCE` for desk usage). |
| Equivalent notional, equivalent ratio | The liquid-tenor notional whose quote sensitivity offsets a position, and its ratio to the position notional; defined in the next lesson (`NEEDS_SOURCE` for desk usage). |
| CDS-bond basis, negative basis trade | The CDS spread minus a bond spread measure; a negative basis trade buys the bond and buys protection. [28] [29] |
| CDS-equivalent bond spread, hazard-adjusted duration | The CDS spread corresponding to the hazard rate that reprices a bond, and a bond duration computed under the hazard model. [30] |
| Cheapest-to-deliver, auction, final price | The delivery option among reference obligations and |

4. Under a fitted piecewise-constant curve, the three-year contract's par spread is 119.94 basis points while its MSQ mark is 120 basis points, and the fit reprices the three-year upfront exactly. Why can both numbers be right?

   (a) They are par spreads under two different survival curves that reprice the same traded upfront with different premium annuities

   (b) The fit failed to reprice the three-year tenor

   (c) The difference is an arbitrage between the two quotations

5. Under the fitted curve, the five-year tenor has protection factor 0.071696769 per unit notional and premium annuity 4.500220799 model-years, and its standard coupon is 100 basis points per year. Calculate the upfront fraction of notional, paid by the protection buyer, that the fit reproduces.

6. A fitted curve has a segment hazard rate of 0.02 per year on (0, 2] and 0.05 per year on (2, 5] model-years. Calculate the survival probability at four years.

*Answers:* *end of the chapter.*

### 7.3.8 Model boundary and review note

This AI-assisted lesson and all cited locators remain `draft` pending human editorial and quantitative review. All input values are invented for teaching. The fitted curve is piecewise constant with knots at quoted tenors, extended flat, under one deterministic discount rate, one deterministic recovery rate, and exact default-time accrual on a regular model-year schedule. A production curve also handles calendar schedules, day counts, business-day rules, settlement timing, accrued premium at inception, an interpolation and extrapolation policy chosen by the trading desk, and a discount curve built from the rates market. The lesson does not model bid-offer, liquidity, dealer inventory, stochastic rates or intensity, recovery risk, counterparty risk, collateral, funding, or transaction costs, and it makes no claim that pricing-model survival probabilities forecast realized defaults.

## References

[1] Shreve, Stochastic Calculus for Finance II: Continuous-Time Models (2004). Ch. 5 §5.2.4, printed pp. 218-219, eqs. 5.2.29-5.2.31.

[2] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.6, printed pp. 368-370.

[3] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.1, printed p. 549 (market-maker bid and offer in basis points per year).

[4] International Swaps and Derivatives Association, ISDA CDS Standard Model and Standard CDS Contract Converter Specification. Standard CDS Contract Converter Specification, version May 5, 2009, Functionality and Specification, printed p. 1. https://www.cdsmodel.com/documentation.html

[5] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.6, printed pp. 366-367 (standard maturity dates on the twentieth of March, June, September, and December; 100 or 500 basis point coupons with a market-determined upfront).

[6] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.4, printed p. 556 (a coupon and a recovery rate are specified for each underlying and maturity).

[7] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.6, printed p. 368, n. 15.

[8] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.6, printed p. 370 (the quote-to-upfront translation is a convention like price and yield; participants need not believe in constant hazard or a particular recovery, but all use the convention).

[9] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.4, printed pp. 556-557 (a hazard rate is implied from the quoted spread by iterative search).

[10] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.2, printed p. 554.

[11] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.7, printed p. 371.

[12] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.2, printed pp. 554-555.

[13] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.5, printed pp. 362-363, Table 14.9 and Figure 14.5.

[14] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.1, printed p. 549.

[15] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.2, printed p. 554, n. 6.

[16] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.4, printed pp. 556-557.

[17] International Swaps and Derivatives Association, ISDA CDS Standard Model and Standard CDS Contract Converter Specification. Standard CDS Examples, version October 26, 2012, printed p. 2 (the converter uses a single flat hazard rate rather than a term structure of flat spreads). https://www.cdsmodel.com/documentation.html

[18] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.1, printed pp. 548-549.

[19] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.5, printed p. 361.

[20] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.5, printed pp. 364-365.

[21] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.5, printed p. 363.

[22] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.5, printed p. 362.

[23] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.6, printed p. 366.

[24] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.4, printed p. 557.

[25] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.10, printed p. 379.

[26] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.6, printed pp. 366-367.

[27] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.3, printed pp. 555-556.

[28] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.8, printed pp. 374-375.

[29] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.1, printed pp. 550-551.

[30] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.7, printed p. 371; §14.9, printed p. 377.

[31] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.1, printed pp. 549 and 551.

[32] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.5, printed p. 363; §14.13, printed p. 387.

[33] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.3, printed p. 555.

[34] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 14 §14.6, printed p. 367, n. 14.

[35] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 24 §24.1, printed p. 550.

## Notation

$\Pi^{\text{buyer}}$ **CDS buyer discounted cash flows**. Random sum of the protection buyer discounted signed cash flows over the contract life before any upfront: protection received on default less scheduled and accrued premium paid, each discounted from its own payment time to valuation time. *Units:* stated currency at valuation time.

$\lambda_k^{\text{flat}}$ **CDS flat tenor hazard rate**. Single constant hazard rate that the standard converter implies for one tenor on its own when it transforms that tenor market-standard quote into an upfront. Different tenors give different flat rates that do not form one term structure. *Units:* decimal intensity per model-year.

$\lambda_k$ **CDS segment hazard rate**. Constant risk-neutral hazard rate on the fitted curve segment between two consecutive tenor maturities. It is one piece of a piecewise-constant term structure, not a separate flat model for that tenor. *Units:* decimal intensity per model-year.

$$\lambda_k \geq 0$$

$K$ **CDS tenor count**. Counts the quoted tenors on the credit curve. It is a positive integer and does not measure time, money, or probability.

$$K \in \{1, 2, ...\}$$

$k$ **CDS tenor index**. Selects one quoted tenor of the credit curve in increasing maturity order. The index is bookkeeping and is not itself a time, a quote, or a probability.

$$k \in \{1, \dots, K\}$$

$T_k$ **CDS tenor maturity**. Maturity of the standard contract quoted at one tenor, measured in model-years from valuation time. Two consecutive tenor maturities bound one hazard segment of the fitted curve. *Units:* model-years from valuation time.

$s_k$ **CDS tenor quote**. Market-standard quote marked at one tenor: the conventional spread that the standard converter turns into that tenor traded upfront using a flat hazard rate for that tenor alone. *Units:* decimal per year in calculations; basis points per year when explicitly quoted.

$u_k$ **CDS tenor upfront fraction**. Signed upfront fraction of notional exchanged at valuation time for the standard contract at one tenor; positive means paid by the protection buyer. It is the traded price that the tenor market-standard quote stands for. *Units:* fraction of notional at valuation time.

$$u_k = U_0/N$$

$A(0,t)$ **Accumulation factor**. Grows one current unit over a stated future horizon under the selected rate model. *Units:* dimensionless currency-units per current currency-unit.

$s_{\mathrm{MSQ}}$ **CDS market-standard quote**. Conventional spread that is the input or the output of the lesson's simplified converter; it equals the zero-upfront par spread within the converter's implied flat-hazard model and need not equal the contract's fixed running coupon. *Units:* decimal per year in calculations; basis points per year when explicitly quoted.

$N$ **CDS notional**. Reference currency amount that scales the simplified premium and protection legs; a positive amount, not itself a signed leg cash flow. *Units:* stated currency.

$s^\star$ **CDS par spread**. Contractual spread that makes the two positive leg magnitudes equal at valuation time with zero upfront amount, solved under the lesson's input curves, recovery, timing, and accrued-premium convention. *Units:* decimal per year in calculations; basis points per year when explicitly quoted.

$A_0^{\mathrm{prem}}$ **CDS premium annuity**. Present-value coefficient that multiplies contractual spread and notional in the simplified premium leg; positive, and includes scheduled premiums and exact accrued premium under the lesson's default-time model. *Units:* model-years of present value per unit notional.

$PV_0^{\mathrm{prem}}$ **CDS premium leg present value**. Positive valuation-time magnitude of the simplified protection buyer's premium payments; its signed contribution to protection-buyer net value is negative. *Units:* stated currency at valuation time.

$PV_0^{\text{prot}}$ **CDS protection leg present value**. Positive valuation-time magnitude of the simplified loss-given-default payment received by the protection buyer after a modeled default. *Units:* stated currency at valuation time.

$c_{\text{std}}$ **CDS standard coupon**. Fixed annualized rate used to determine the contract's running premium cash flows in the lesson's standard-coupon model; a positive rate paid by the protection buyer on surviving notional and as accrued premium after default. *Units:* decimal per year in calculations; basis points per year when explicitly quoted.

$U_0$ **CDS upfront amount**. Time-zero cash amount that balances protection and fixed-coupon premium value under the lesson's pricing convention; positive means paid by the protection buyer and negative means received by the protection buyer. *Units:* stated currency at valuation time.

$m$ **Compounding frequency**. Number of equal compounding periods per year under the stated rate convention, a positive integer fixed by the model convention. *Units:* compounding periods per year.

$D(0,t)$ **Discount factor**. Converts one deterministic future unit into value at valuation time. *Units:* current currency-units per future currency-unit.

$\mathbb{E}$ **Expectation**. Operator returning the average of a random quantity, each outcome weighted by its probability under a stated measure; a superscript names that measure when more than one is in play. *Units:* value units.

$$\mathbb{E}[X] = \int_{\Omega} X(\omega)\, d\mathbb{P}(\omega)$$

$\lambda$ **Constant hazard rate**. Constant conditional default intensity used by the lesson's simplified exponential survival model; a risk-neutral pricing input conditional on survival to the current instant, not a cumulative probability. *Units:* decimal intensity per model-year.

LGD **Loss given default**. Fraction of an explicitly stated reference amount not recovered under a deterministic recovery convention, relative to the same reference amount used by the recovery rate. *Units:* decimal fraction between zero and one.

$j^{(m)}$ **Nominal annual rate**. Annualized rate quote that must be paired with its compounding frequency. *Units:* decimal rate per year.

$t_k$ **Payment time**. Time from the valuation date to one scheduled cash-flow event; a time coordinate, not a calendar date or payment index. *Units:* years from the valuation date.

$r_m$ **Periodic rate**. Rate applied once in each compounding period under the stated convention, derived from the stated nominal annual quote in this model. *Units:* decimal per compounding period.

$PV_0$ **Present value**. Combines dated signed cash flows into one value at valuation time, using the same holder perspective as the signed cash flows. *Units:* stated currency at the valuation time.

$R$ **Recovery rate**. Fraction of a stated reference amount recovered after a modeled default under an explicitly stated recovery convention; a non-negative fraction whose reference amount, payment timing, and settlement convention are set by the model that uses it. *Units:* decimal fraction between zero and one.

$\mathbb{Q}$ **Risk-neutral probability measure**. Gives model pricing weights under which discounted traded prices satisfy the martingale condition; it prices payoffs relative to a stated numeraire and is not a forecast of actual event frequencies. *Units:* dimensionless probability weights between zero and one.

$CF_k$ **Signed cash flow**. Amount received or paid at one event from the stated holder perspective; positive means received and negative means paid by that holder. *Units:* stated currency units.

$S(0,t)$ **Survival probability**. Probability, under the explicitly stated model measure, that no modeled default has occurred between valuation time and a stated future time. *Units:* probability between zero and one.

0 **Valuation time**. Common origin from which later model times and present values are measured. *Units:* years from the valuation date.

## Answers to the knowledge checks

**Knowledge check 7.3.1: Credit curve and market observables**

1. **(a)** The flat risk-neutral hazard rate. The coupon is contractual and the recovery rate is a convention assumed by the converter. The hazard rate is the unknown that the one traded price identifies through the equality of expected discounted leg values.

2. **(a)** The 40% recovery is a convention assumed by the converter; the quote does not observe it. One traded price cannot identify both recovery and default timing, so the converter assumes recovery and solves the hazard rate. The bid and offer are the observed market levels; the recovery is not among them.

3. **(a)** A change of quotation using a separate flat hazard rate per tenor; it does not build one term structure. The tenor-by-tenor conversion is a curve transformation. Each tenor's flat hazard rate answers a separate question, so the three flat rates are not one survival curve; fitting the curve is a separate step that solves segment hazards in maturity order.

4. **(a)** They are par spreads under two different survival curves that reprice the same traded upfront with different premium annuities. The MSQ mark is the par spread under the converter's single flat hazard for that tenor; the fitted par spread uses the term structure's premium annuity. Both reproduce the same traded upfront, and the hypothetical zero-upfront contract is not itself traded, so there is no arbitrage.

5. 0.026694561 (accepted within $\pm 0.000001$). The fit condition per unit notional is protection factor minus coupon times premium annuity: $0.071696769 - 0.010 \times 4.500220799 = 0.026694561$, which equals the five-year mark.

6. 0.869358235 (accepted within $\pm 0.000001$). Survival compounds segment by segment: $\exp(-0.02 \times 2) \times \exp(-0.05 \times 2) = \exp(-0.14) = 0.869358235$. Using only the second segment's rate over four years would be wrong.