Chapter 2.3

Yield to maturity as a single-rate summary

From *Credit Products Playground*, with changes after version 0.4.0, not yet released. Interactive version: https://danieltuzes.github.io/credit-and-derivatives/bonds/yield-to-maturity/

2.3.1 What you will be able to do

After this lesson, you should be able to:

- interpret yield to maturity as a model-dependent single-rate summary;
- state its annualization and compounding convention;
- calculate the price from an input yield without changing the promised cash flows.

2.3.2 What yield means in this toy model

Use the bond-payment-index k to select one of the number-of-bond-payments n, and write its promised bond-cash-flow as CF_k^{bond}. Write the bond-price as P_0 and the bond-payment-frequency as m_{B}.

Write yield-to-maturity as $y^{(m_{\text{B}})}$. It is the one nominal annual rate, compounded at the bond payment frequency, that makes the discounted promised payments equal the bond price:

$$P_0 = \sum_{k=1}^{n} \frac{CF_k^{\text{bond}}}{\left(1 + y^{(m_{\text{B}})}/m_{\text{B}}\right)^k} \tag{2.3.1}$$

The symbol $y^{(m_{\text{B}})}$ could be mistaken for a spot rate, an effective annual rate, an expected return, or another quote. In this course it means only the bond yield defined above.

2.3.3 Coupon rate and yield have different roles

- The coupon rate determines the promised coupon-payment C in the contract.
- The input yield determines the denominators in the price formula.

A change of the input yield does not change the coupons or the face-value F of the bond.

Example 2.3.1: Pricing from a yield to maturity

Annual payments

A two-year bond with an annual coupon rate of 5% and a face value of USD 1,000 pays USD 50 and USD 1,050. At an annual yield of 6% with annual compounding, the price is:

$$P_0 = \frac{50}{1.06} + \frac{1050}{1.06^2} \approx \text{USD } 981.67$$

Semiannual compounding

For the same face value and coupon rate over two years, but with semiannual payments, $C = 25$, $m_{\mathrm{B}} = 2$, and $n = 4$. A nominal annual yield of 6% gives a periodic rate of 3%, and the price is:

$$P_0 = \frac{25}{1.03} + \frac{25}{1.03^2} + \frac{25}{1.03^3} + \frac{1025}{1.03^4} \approx \text{USD } 981.41$$

The annual and the semiannual example use the same yield number, 6%, but different payment times and different compounding.

Zero-coupon special case

With no coupons and maturity-time T, the price is:

$$P_0 = \frac{F}{\left(1 + y^{(m_{\mathrm{B}})}/m_{\mathrm{B}}\right)^{m_{\mathrm{B}}T}}$$

For USD 1,000 due in three years at 4% with annual compounding, the price is approximately USD 889.00.

2.3.4 Check your understanding

These items separate yield interpretation from annual and periodic-compounding calculations. Each question stays collapsed until you open it; answers and explanations appear once you check.

Knowledge check 2.3.1: Yield to maturity

1. In the simplified lesson model, what is yield to maturity?

   (a) The single nominal annual rate, compounded at coupon frequency, that reproduces price from promised cash flows

   (b) An additional cash flow paid by the issuer at maturity

   (c) The currency amount of each coupon

2. Two bonds have different promised cash-flow amounts and maturities, but the same coupon frequency and the same quoted yield to maturity under the simplified nominal convention. Which statement follows?

   (a) The same summary rate is used, but each bond's own cash flows still determine its price

(b) The two bonds must have the same price

(c) The two bonds must have identical coupon cash flows

3. Price in USD a two-year bond with USD 1,000 face value, a 5% annual coupon rate, and one coupon payment per year at a 6% nominal annual yield compounded annually. Assume valuation on a coupon date and no accrued interest.

4. Price in USD a two-year bond with USD 1,000 face value, a 5% annual coupon rate, and two coupon payments per year at a 6% nominal annual yield compounded semiannually. Assume valuation on a coupon date and no accrued interest.

Answers: *end of the chapter.*

2.3.5 Model boundary and review note

This lesson calculates the price from an input yield. It does not solve numerically for the yield from a price. A real bond implementation also needs schedules, day counts, accrued interest, clean and dirty prices, and possibly credit and option models.

The single-rate definition and the $2T$-period pricing formula follow Tuckman & Serrat[1]; Hull gives the same definition, solved iteratively[2]. FINRA defines it as the return to an investor who buys at the market price and holds to maturity[3]. The lesson stays `draft` pending human confirmation of the printed locators.

References

[1] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). §3.2, "Yield to Maturity": the single rate that discounts a bond's cash flows to its market price (eq. 3.5 and the general formula), also its internal rate of return.

[2] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 4, "Bond Yield".

[3] FINRA, Understanding Bond Yield and Return. "Yield to maturity" section. https://www.finra.org/investors/insights/bond-yield-return

Notation

$A(0,t)$ **Accumulation factor**. Grows one current unit over a stated future horizon under the selected rate model. *Units:* dimensionless currency-units per current currency-unit.

c **Annual coupon rate**. Contractual annual rate used to determine a fixed-rate bond's coupon payments; applied to face value, not to market price. *Units:* decimal rate per year.

CF_k^{bond} **Bond cash flow**. Promised amount paid to the bondholder on one scheduled payment date; a positive receipt for the bondholder, with default excluded in the bond lessons. *Units:* stated currency at payment time.

m_{B} **Bond payment frequency**. Number of scheduled coupon payments per year in the simplified regular bond, a positive integer. *Units:* scheduled coupon payments per year.

k **Bond payment index**. Labels one remaining scheduled bond payment in increasing time order; it selects a payment and is not itself a time or currency amount. *Units:* dimensionless schedule index.

P_0 **Bond price**. Present value of the simplified bond's promised payments at valuation time; the amount paid by the buyer, shown as a positive value in the bond lessons. *Units:* stated currency at valuation time.

m **Compounding frequency**. Number of equal compounding periods per year under the stated rate convention, a positive integer fixed by the model convention. *Units:* compounding periods per year.

C **Coupon payment**. Level periodic cash amount promised by the simplified fixed-rate bond; a positive receipt for the bondholder in these lessons. *Units:* stated currency per coupon date.

$D(0,t)$ **Discount factor**. Converts one deterministic future unit into value at valuation time. *Units:* current currency-units per future currency-unit.

F **Face value**. Contractual reference amount used to determine coupons and principal redemption. *Units:* stated currency.

T **Maturity time**. Final scheduled time when principal is redeemed in the simplified bond. *Units:* model-years from the valuation date.

$j^{(m)}$ **Nominal annual rate**. Annualized rate quote that must be paired with its compounding frequency. *Units:* decimal rate per year.

n **Number of bond payments**. Counts the remaining regular coupon dates including maturity; a positive integer for the simplified regular bond schedule. *Units:* scheduled payment dates.

t_k **Payment time**. Time from the valuation date to one scheduled cash-flow event; a time coordinate, not a calendar date or payment index. *Units:* years from the valuation date.

r_m **Periodic rate**. Rate applied once in each compounding period under the stated convention, derived from the stated nominal annual quote in this model. *Units:* decimal per compounding period.

PV_0 **Present value**. Combines dated signed cash flows into one value at valuation time, using the same holder perspective as the signed cash flows. *Units:* stated currency at the valuation time.

CF_k **Signed cash flow**. Amount received or paid at one event from the stated holder perspective; positive means received and negative means paid by that holder. *Units:* stated currency units.

0 **Valuation time**. Common origin from which later model times and present values are measured. *Units:* years from the valuation date.

$y^{(m_{\mathrm{B}})}$ **Yield to maturity**. Single nominal annual rate that reproduces the simplified bond price. *Units:* nominal annual decimal rate compounded at the bond payment frequency.

Answers to the knowledge checks

Knowledge check 2.3.1: Yield to maturity

1. **(a)** The single nominal annual rate, compounded at coupon frequency, that reproduces price from promised cash flows. The toy model defines yield to maturity as one rate that discounts all promised cash flows to the stated price under the stated compounding convention.

2. **(a)** The same summary rate is used, but each bond's own cash flows still determine its price. A common yield does not erase differences in amount or timing; the model discounts each bond's own promised cash flows.

3. 981.6660733357066 (accepted within ± 0.01). $P_0 = \frac{50}{1.06} + \frac{1{,}050}{1.06^2}$.

4. 981.4145079859479 (accepted within ± 0.01). Each coupon is USD 25 and the periodic yield is $0.06/2 = 0.03$, so $P_0 = \frac{25}{1.03} + \frac{25}{1.03^2} + \frac{25}{1.03^3} + \frac{1{,}025}{1.03^4}$.