Chapter 2.5

Settlement, accrued interest, and clean versus dirty price

From *Credit Products Playground*, with changes after version 0.4.0, not yet released. Interactive version: https://danieltuzes.github.io/credit-and-derivatives/bonds/settlement-clean-and-dirty-price/

2.5.1 The price paid is not always the price quoted

A bond price quote commonly shows the clean-bond-price, also called the flat or quoted price. The buyer pays the dirty-bond-price, also called the full, cash, or invoice price. The dirty price is the clean price plus the accrued-interest:

$$P^{\text{dirty}} = P^{\text{clean}} + AI \tag{2.5.1}$$

On a coupon date, the accrued interest is zero, so the two prices are equal. For this reason, the earlier bond lessons, which settle on a coupon date, use one bond-price symbol.

2.5.2 Actual/actual within one coupon period

Under the convention of this lesson, let the local coupon-period-elapsed-days be n_{elapsed} and the local coupon-period-total-days be n_{period}. For one coupon-payment C, the accrued interest is the coupon multiplied by the fraction of the coupon period that has elapsed:

$$AI = C\frac{n_{\text{elapsed}}}{n_{\text{period}}} \tag{2.5.2}$$

Consider the coupon period from February 15, 2021 to August 15, 2021, which has 181 days. Settlement on May 17, 2021 is 91 actual days after the start of the period. With a coupon of USD 31.25, (2.5.2) gives the accrued interest:

$$AI = 31.25\left(\frac{91}{181}\right) = 15.71132597 \text{ USD per bond} \tag{2.5.3}$$

The domain code checks that the dates are valid ISO dates and counts UTC calendar days. It is not a production schedule engine: business-day rules, ex-coupon periods, stubs, and other day counts require separate conventions.

2.5.3 Why this matters for derivatives

The cash amount paid at the settlement of a bond forward or a bond option depends on whether its price is clean or dirty. If a strike is quoted as a clean price and the modeled bond price is a dirty price, add the accrued interest at the exercise date to the strike before the payoff is calculated. Also, each coupon paid before delivery must be assigned to the party that owns the bond on the coupon date.

Knowledge check 2.5.1: Bond settlement

1. A coupon payment is USD 31.25. Settlement is 91 actual days into an actual 181-day coupon period. Calculate accrued interest in USD.
2. A bond has a clean cash price of USD 9,178.125 and accrued interest of USD 15.711. Calculate its dirty invoice price in USD.

Answers: *end of the chapter.*

2.5.4 Sources

The identity between the clean and the dirty price (2.5.1) and the actual/actual calculation (2.5.2) follow Tuckman and Serrat, Chapter 1 §1.6 [1]. Hull's European bond option discussion emphasizes matching cash versus quoted strike and bond prices [2].

References

[1] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022).

[2] Hull, Options, Futures, and Other Derivatives (8th ed., 2012).

Notation

n_{elapsed} **Coupon period elapsed days**. Actual calendar days from the previous coupon date through settlement under the lesson convention. *Units:* days.

n_{period} **Coupon period total days**. Actual calendar days from the previous coupon date to the next coupon date. *Units:* days.

AI **Accrued interest**. Coupon amount attributed to the interval from the previous coupon date through settlement under the stated day-count convention; a positive amount added to the clean price to obtain the dirty invoice price. *Units:* stated currency at settlement.

c **Annual coupon rate**. Contractual annual rate used to determine a fixed-rate bond's coupon payments; applied to face value, not to market price. *Units:* decimal rate per year.

m_{B} **Bond payment frequency**. Number of scheduled coupon payments per year in the simplified regular bond, a positive integer. *Units:* scheduled coupon payments per year.

P^{clean} **Clean bond price**. Bond price quoted without accrued interest under the stated settlement convention; a positive quoted price to the buyer before accrued interest is added. *Units:* stated currency at settlement.

m **Compounding frequency**. Number of equal compounding periods per year under the stated rate convention, a positive integer fixed by the model convention. *Units:* compounding periods per year.

C **Coupon payment**. Level periodic cash amount promised by the simplified fixed-rate bond; a positive receipt for the bondholder in these lessons. *Units:* stated currency per coupon date.

P^{dirty} **Dirty bond price**. Full cash or invoice price paid for the bond, equal to clean price plus accrued interest; a positive cash price paid by the buyer under the stated settlement convention. *Units:* stated currency at settlement.

F **Face value**. Contractual reference amount used to determine coupons and principal redemption. *Units:* stated currency.

Answers to the knowledge checks

Knowledge check 2.5.1: Bond settlement

1. 15.71132596685083 (accepted within ± 0.000001). Accrued interest is $31.25 \times 91 / 181 =$ USD 15.71132597.

2. 9193.836 (accepted within ± 0.000001). Dirty price = clean price + accrued interest = USD 9,193.836.