Chapter 2.4

The bond price-yield relationship

From *Credit Products Playground*, with changes after version 0.4.0, not yet released. Interactive version: https://danieltuzes.github.io/credit-and-derivatives/bonds/price-yield-relationship/

2.4.1 What you will be able to do

After this lesson, you should be able to:

- explain why the price of a bond with fixed positive cash flows decreases when the yield increases;
- hold contract cash flows fixed while changing only the discounting input;
- recognize that the price-yield relationship is curved rather than linear.

2.4.2 The price decreases because every denominator increases

Write the bond-payment-frequency as m_{B}. In this lesson, y denotes the yield-to-maturity $y^{(m_{\mathrm{B}})}$ without its superscript, as the argument of a function. Write the price-yield-curve as $P(y)$.

Use the bond-payment-index k to select one of the number-of-bond-payments n, and write each promised positive bond-cash-flow as CF_k^{bond}. The price at yield y is the sum of the promised cash flows, each discounted at the yield:

$$P(y) = \sum_{k=1}^{n} \frac{CF_k^{\mathrm{bond}}}{\left(1 + y/m_{\mathrm{B}}\right)^k} \tag{2.4.1}$$

Assume that the yield stays non-negative and that every cash flow stays fixed. If the yield increases, every denominator increases. So every discounted cash flow decreases, and the bond-price decreases. Table 2.4.1 also assumes that the new yield is non-negative.

Define the local yield-shock Δy as new yield minus old yield, and define the local price-change ΔP as new price minus old price.

The table holds only if the promised cash flows stay fixed.

Example 2.4.1: Price-yield comparisons

Three points form a curve

Consider a five-year bond with an annual coupon rate of 5%, semiannual payments, and a face

Table 2.4.1 **Signs of a yield shock and the price change.** For a rise and a fall in the yield, the direction of every discount denominator and the sign of the price change, with the promised cash flows held fixed.

Yield shock	Denominators	Price change
$\Delta y > 0$	Increase	$\Delta P < 0$
$\Delta y < 0$	Decrease	$\Delta P > 0$

value of 100. Its prices at three yields are:

Nominal annual yield	Price per 100 face
5%	100.0000
6%	95.7349
7%	91.6834

The price decreases as the yield increases. Each yield increase is one percentage point, but the two price changes are different: 4.2651 and 4.0515. So the three points are not on a straight line.

Equal shocks, unequal price changes

Start from a yield of 5%:

- a yield decrease to 4% increases the price from 100.0000 to 104.4913, a change of +4.4913;
- a yield increase to 6% decreases the price from 100.0000 to 95.7349, a change of −4.2651.

The two yield shocks have equal size, but the absolute price changes are different. This difference is a result of the curvature of the price-yield curve. This lesson does not calculate duration, DV01, or convexity.

Hold other inputs fixed

Suppose the issuer changes a coupon, a default occurs, or the settlement date changes. Then more than the yield has changed, and the comparison no longer isolates the effect of the yield.

One tested domain function calculates every price in Figure 2.4.1. The notation panel only explains the symbols; it does not calculate prices.

2.4.3 Check your understanding

These direct and transfer items cover direction and nonlinearity. Each question stays collapsed until you open it; answers and explanations appear once you check.

Knowledge check 2.4.1: Bond price-yield shape

1. Within the lesson's non-negative-yield domain, all else equal, what happens to a fixed bond's price when its yield rises?

   (a) The price falls

   (b) The price rises

   (c) The price is unchanged

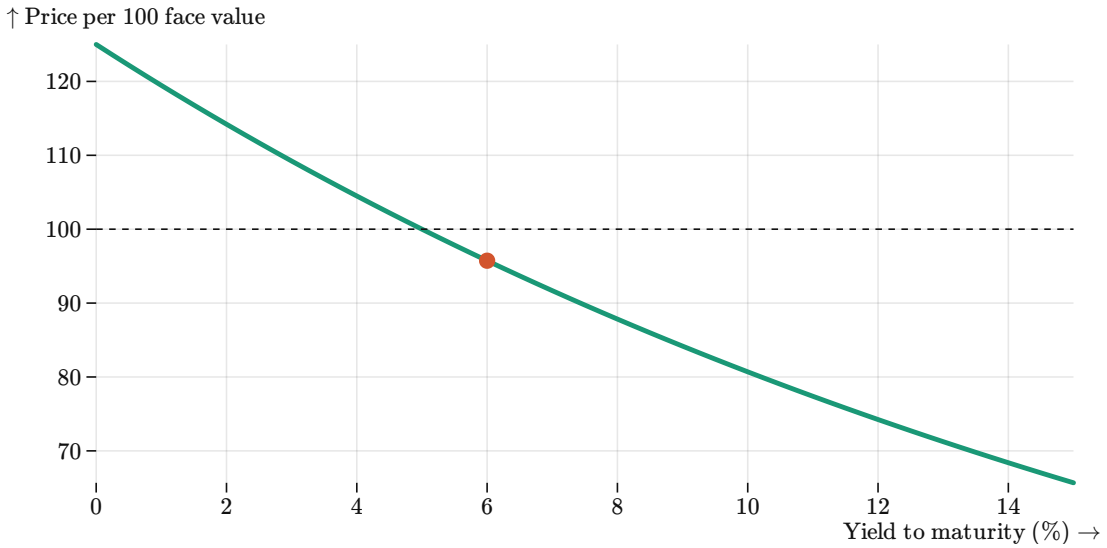


Figure 2.4.1 **Bond price–yield explorer.** The price per 100 face value of one fixed-coupon bond at every yield to maturity from 0% to 15%. The dot marks the price at the yield shown; the dashed line marks par, a price of 100. Shown for a 5% annual coupon rate, 2 coupon payments a year and a 5-year maturity: at a 6% yield to maturity, the price is 95.73 per 100 face value.

2. A bond's yield falls from 6% to 5% while promised cash flows stay fixed. Which explanation is correct?

   (a) Discounting denominators shrink, so price rises

   (b) The contractual coupons grow, so price rises

   (c) The principal shrinks, so price falls

3. A fixed bond's prices at yields of 4%, 5%, and 6% are 103.0, 100.0, and 97.2. What do the unequal changes for equal one-percentage-point yield steps show?

   (a) The price-yield relationship is not a straight line

   (b) Price is independent of yield

   (c) The promised cash flows must change with yield

4. For three equally spaced yields, the middle bond price differs from the average of the two endpoint prices. What is the best interpretation?

   (a) The sampled price-yield relationship has curvature

   (b) The sampled price-yield relationship is exactly linear

   (c) Yield has no relationship to price

Answers: *end of the chapter.*

2.4.4 Model boundary and review note

The plotted curve summarizes a toy model with one yield. It is not a credit curve and not a recommendation. It omits accrued interest, term-structure effects, default, recovery, liquidity, tax, funding, and embedded options.

The inverse direction and the curvature both follow Tuckman & Serrat[1]; Hull notes the same negative price-yield relationship[2]. FINRA states that price and yield are inversely related[3]. The lesson stays `draft` pending human confirmation of the printed locators.

References

[1] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). §4.1, "Price-Rate Curves", Figure 4.2, prices decrease as rates rise; §4.5, "Convexity", price-rate curves of coupon bonds are convex.

[2] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). §4.8, Duration.

[3] FINRA, Understanding Bond Yield and Return. "Price and yield" section. https://www.finra.org/investors/insights/bond-yield-return

Notation

ΔP **Price change**. New toy-model price minus old toy-model price for a stated yield shock. This finite change is not yet a duration or convexity approximation. *Units:* stated currency per bond at valuation time.

$$\Delta P = P(y + \Delta y) - P(y)$$

Δy **Yield shock**. A lesson-local finite change in the input yield. The sign is new yield minus old yield; the unit must be decimal, percentage points, or basis points as explicitly labeled. *Units:* decimal rate per year.

$$\Delta y = y_{\text{new}} - y_{\text{old}}$$

$A(0,t)$ **Accumulation factor**. Grows one current unit over a stated future horizon under the selected rate model. *Units:* dimensionless currency-units per current currency-unit.

c **Annual coupon rate**. Contractual annual rate used to determine a fixed-rate bond's coupon payments; applied to face value, not to market price. *Units:* decimal rate per year.

CF_k^{bond} **Bond cash flow**. Promised amount paid to the bondholder on one scheduled payment date; a positive receipt for the bondholder, with default excluded in the bond lessons. *Units:* stated currency at payment time.

m_{B} **Bond payment frequency**. Number of scheduled coupon payments per year in the simplified regular bond, a positive integer. *Units:* scheduled coupon payments per year.

k **Bond payment index**. Labels one remaining scheduled bond payment in increasing time order; it selects a payment and is not itself a time or currency amount. *Units:* dimensionless schedule index.

P_0 **Bond price**. Present value of the simplified bond's promised payments at valuation time; the amount paid by the buyer, shown as a positive value in the bond lessons. *Units:* stated currency at valuation time.

m **Compounding frequency**. Number of equal compounding periods per year under the stated rate convention, a positive integer fixed by the model convention. *Units:* compounding periods per year.

C **Coupon payment**. Level periodic cash amount promised by the simplified fixed-rate bond; a positive receipt for the bondholder in these lessons. *Units:* stated currency per coupon date.

$D(0,t)$ **Discount factor**. Converts one deterministic future unit into value at valuation time. *Units:* current currency-units per future currency-unit.

F **Face value**. Contractual reference amount used to determine coupons and principal redemption. *Units:* stated currency.

T **Maturity time**. Final scheduled time when principal is redeemed in the simplified bond. *Units:* model-years from the valuation date.

$j^{(m)}$ **Nominal annual rate**. Annualized rate quote that must be paired with its compounding frequency. *Units:* decimal rate per year.

n **Number of bond payments**. Counts the remaining regular coupon dates including maturity; a positive integer for the simplified regular bond schedule. *Units:* scheduled payment dates.

t_k **Payment time**. Time from the valuation date to one scheduled cash-flow event; a time coordinate, not a calendar date or payment index. *Units:* years from the valuation date.

r_m **Periodic rate**. Rate applied once in each compounding period under the stated convention, derived from the stated nominal annual quote in this model. *Units:* decimal per compounding period.

PV_0 **Present value**. Combines dated signed cash flows into one value at valuation time, using the same holder perspective as the signed cash flows. *Units:* stated currency at the valuation time.

$P(y)$ **Price-yield curve**. Bond price as a function of yield while promised positive fixed cash flows remain constant and only the yield varies. *Units:* stated currency per bond.

CF_k **Signed cash flow**. Amount received or paid at one event from the stated holder perspective; positive means received and negative means paid by that holder. *Units:* stated currency units.

0 **Valuation time**. Common origin from which later model times and present values are measured. *Units:* years from the valuation date.

$y^{(m_B)}$ **Yield to maturity**. Single nominal annual rate that reproduces the simplified bond price. *Units:* nominal annual decimal rate compounded at the bond payment frequency.

Answers to the knowledge checks

Knowledge check 2.4.1: Bond price-yield shape

1. **(a)** The price falls. A higher yield produces smaller discount factors for every positive future cash flow.

2. **(a)** Discounting denominators shrink, so price rises. The promised coupons do not change; the change comes from discounting them at a lower yield.

3. **(a)** The price-yield relationship is not a straight line. The first equal yield step changes price by 3.0 while the second changes it by 2.8, so the slope is not constant.

4. **(a)** The sampled price-yield relationship has curvature. For a straight line, the value at the midpoint equals the average of endpoint values; a difference is evidence of curvature.