Chapter 2.2

Bond price from discount factors

From *Credit Products Playground*, with changes after version 0.4.0, not yet released. Interactive version: https://danieltuzes.github.io/credit-and-derivatives/bonds/price-from-discount-factors/

2.2.1 What you will be able to do

After this lesson, you should be able to calculate a toy bond's price from its promised cash-flow schedule and input discount factors.

2.2.2 Bond pricing is a present-value application

Use the bond-payment-index k to select one of the number-of-bond-payments n. Write the promised bond-cash-flow as CF_k^{bond}, its payment-time as t_k, and its matching discount-factor as $D(0, t_k)$. Write the bond-price as P_0. The bond price is the present-value at valuation-time 0 of the promised payments. So the bond price is the schedule sum (1.5.1), applied to the promised payments of a bond:

$$P_0 = \sum_{k=1}^{n} CF_k^{\text{bond}} D(0, t_k) \tag{2.2.1}$$

In this lesson, "price" means this present value. It does not mean a clean price, a quoted price, or a probability. Those concepts are outside this lesson.

Example 2.2.1: Bond prices from discount factors

Two discount factors

The annual-coupon bond from the preceding lesson pays USD 50 after one year and USD 1,050 after two years. Suppose the matching discount factors are 0.96 and 0.91.

Payment index k	Cash flow, USD	Discount factor	Present value, USD
1	50	0.96	48.00
2	1,050	0.91	955.50

The bond price is the sum of the present values:

$$P_0 = 50(0.96) + 1050(0.91) = \text{USD } 1{,}003.50$$

The price is above USD 1,000 because, with these two discount factors, the present value of the promised payments is larger than the face value. This lesson does not use a yield.

Unfamiliar schedule

Consider payments of USD 20 at 0.5 years, USD 20 at 1.0 year, and USD 1,020 at 1.5 years, with discount factors 0.98, 0.95, and 0.92. The bond price is:

$$P_0 = 20(0.98) + 20(0.95) + 1020(0.92) = 19.60 + 19.00 + 938.40 = \text{USD } 977.00$$

Zero-coupon boundary

If every coupon is zero, the only payment is the face-value F at maturity-time T. The bond price is then:

$$P_0 = FD(0,T)$$

This is (2.2.1) with one payment. It is not a separate rule.

2.2.3 Error pattern: discount after adding

For the two-payment example, compare a wrong calculation, which applies one discount factor to the sum of the payments, with the calculation in (2.2.1):

$$\begin{aligned} (50 + 1050)(0.91) &= 1001.00 \\ 50(0.96) + 1050(0.91) &= 1003.50 \\ 1001.00 &\neq 1003.50 \end{aligned} \tag{2.2.2}$$

One common discount factor is correct only when the model assigns that discount factor to every payment.

2.2.4 Check your understanding

This set includes one coupon-bond calculation and one transfer schedule. Each question stays collapsed until you open it; answers and explanations appear once you check.

Knowledge check 2.2.1: Bond price from discount factors

1. From the holder's perspective, a simplified bond pays USD +50 at t = 1 year and USD +1,050 at t = 2 years. The matching discount factors are 0.95 and 0.90. What is its time-zero price in USD?

2. From the holder's perspective, a simplified bond pays USD +20, USD +20, and USD +1,020 at three successive dates. Their matching discount factors are 0.98, 0.95, and 0.91. What is its time-zero price in USD?

Answers: end of the chapter.

2.2.5 Model boundary and review note

The discount factors are inputs; they are not calibrated. This lesson has no credit risk, although the course is about credit products.

The pricing rule (2.2.1) follows Tuckman & Serrat[1]; Hull discounts each cash flow at its own zero rate the same way[2]. The lesson stays `draft` pending human confirmation of the printed locators.

References

[1] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). §1.2, each bond price as its cash flows times the matching discount factors (eqs. 1.1-1.3), checked against market prices in Table 1.4.

[2] Hull, Options, Futures, and Other Derivatives (8th ed., 2012). Ch. 4, "Bond Pricing".

Notation

$A(0,t)$ **Accumulation factor**. Grows one current unit over a stated future horizon under the selected rate model. *Units:* dimensionless currency-units per current currency-unit.

c **Annual coupon rate**. Contractual annual rate used to determine a fixed-rate bond's coupon payments; applied to face value, not to market price. *Units:* decimal rate per year.

CF_k^{bond} **Bond cash flow**. Promised amount paid to the bondholder on one scheduled payment date; a positive receipt for the bondholder, with default excluded in the bond lessons. *Units:* stated currency at payment time.

m_{B} **Bond payment frequency**. Number of scheduled coupon payments per year in the simplified regular bond, a positive integer. *Units:* scheduled coupon payments per year.

k **Bond payment index**. Labels one remaining scheduled bond payment in increasing time order; it selects a payment and is not itself a time or currency amount. *Units:* dimensionless schedule index.

P_0 **Bond price**. Present value of the simplified bond's promised payments at valuation time; the amount paid by the buyer, shown as a positive value in the bond lessons. *Units:* stated currency at valuation time.

m **Compounding frequency**. Number of equal compounding periods per year under the stated rate convention, a positive integer fixed by the model convention. *Units:* compounding periods per year.

C **Coupon payment**. Level periodic cash amount promised by the simplified fixed-rate bond; a positive receipt for the bondholder in these lessons. *Units:* stated currency per coupon date.

$D(0,t)$ **Discount factor**. Converts one deterministic future unit into value at valuation time. *Units:* current currency-units per future currency-unit.

F **Face value**. Contractual reference amount used to determine coupons and principal redemption. *Units:* stated currency.

T **Maturity time**. Final scheduled time when principal is redeemed in the simplified bond. *Units:* model-years from the valuation date.

$j^{(m)}$ **Nominal annual rate**. Annualized rate quote that must be paired with its compounding frequency. *Units:* decimal rate per year.

n **Number of bond payments**. Counts the remaining regular coupon dates including maturity; a positive integer for the simplified regular bond schedule. *Units:* scheduled payment dates.

t_k **Payment time**. Time from the valuation date to one scheduled cash-flow event; a time coordinate, not a calendar date or payment index. *Units:* years from the valuation date.

r_m **Periodic rate**. Rate applied once in each compounding period under the stated convention, derived from the stated nominal annual quote in this model. *Units:* decimal per compounding period.

PV_0 **Present value**. Combines dated signed cash flows into one value at valuation time, using the same holder perspective as the signed cash flows. *Units:* stated currency at the valuation time.

CF_k **Signed cash flow**. Amount received or paid at one event from the stated holder perspective; positive means received and negative means paid by that holder. *Units:* stated currency units.

0 **Valuation time**. Common origin from which later model times and present values are measured. *Units:* years from the valuation date.

Answers to the knowledge checks

Knowledge check 2.2.1: Bond price from discount factors

1. 992.5 (accepted within $\pm 1e-9$). Price $= 50 \times 0.95 + 1{,}050 \times 0.90 = 47.5 + 945 = 992.5$.
2. 966.8 (accepted within $\pm 1e-9$). Price $= 20 \times 0.98 + 20 \times 0.95 + 1{,}020 \times 0.91 = 19.6 + 19 + 928.2 = 966.8$.