Chapter 2.1

Fixed-rate bond contract and cash flows

From *Credit Products Playground*, with changes after version 0.4.0, not yet released. Interactive version: https://danieltuzes.github.io/credit-and-derivatives/bonds/fixed-rate-contract-and-cash-flows/

2.1.1 What you will be able to do

After this lesson, you should be able to:

- identify the terms that determine a toy fixed-rate bond's payments;
- calculate the level coupon amount;
- construct the complete promised cash-flow schedule.

2.1.2 What a bond is

A bond is a debt claim. The issuer is the borrower; the bondholder is the lender who owns the claim. In the fixed-rate example used here, the issuer promises a sequence of coupon payments and repayment of principal at maturity. The price paid for the bond is a different quantity from the promised cash flows.

The word "promises" is important. A contract can specify an amount without a guarantee that the issuer pays it. This first bond lesson constructs the contractual schedule. The credit-risk lessons introduce default probability and recovery later. Tuckman and Serrat define a coupon bond by its coupon rate, its maturity, and its face amount (also called the par amount or the principal amount). They show the coupons and the principal on the cash-flow timeline[1].

2.1.3 Read the contract before valuing it

Write the simplified contract's face-value as F, its annual-coupon-rate as c, its bond-payment-frequency as m_{B}, and its maturity-time as T. Write the number-of-bond-payments as n and index those dates with the bond-payment-index k. The matching payment-time is t_k, measured from valuation-time 0.

Write the level coupon-payment as C and the promised bond-cash-flow on date k as CF_k^{bond}.

The number of payment dates is the payment frequency multiplied by the maturity:

$$n = m_{\mathrm{B}} T \tag{2.1.1}$$

The bond payment frequency is a different concept from the compounding-frequency m of the earlier lessons. So it has a different symbol, with the subscript B.

2.1.4 Calculate the coupon

The level coupon payment is the annual coupon rate multiplied by the face value, divided by the payment frequency:

$$C = \frac{cF}{m_{\mathrm{B}}} \tag{2.1.2}$$

The coupon rate is applied to the face value, not to the market price of the bond.

2.1.5 Construct every promised payment

For the bondholder, the promised bond cash flow on date k is the coupon, plus the face value on the final date:

$$CF_k^{\mathrm{bond}} = C + \mathbf{1}_{\{k=n\}} F \tag{2.1.3}$$

The indicator $\mathbf{1}_{\{k=n\}}$ equals one on the final date and zero on every other date. So the face value is paid only on the final date.

On the regular schedule, the payment time of date k, measured from valuation time, is:

$$t_k = \frac{k}{m_{\mathrm{B}}}, \qquad 0 \text{ is the valuation time} \tag{2.1.4}$$

Example 2.1.1: Coupon schedules by payment frequency

Annual coupons

A two-year bond has a face value of USD 1,000, an annual coupon rate of 5%, and one payment per year. The coupon is:

$$C = \frac{0.05 \times 1000}{1} = \text{USD } 50$$

Payment index k	Payment time t_k, years	Coupon, USD	Principal, USD	Total, USD
1	1.0	50	0	50
2	2.0	50	1,000	1,050

Semiannual coupons

An 18-month bond has a face value of USD 1,000, an annual coupon rate of 4%, and two payments per year. The coupon and the number of payments are:

$$C = \frac{0.04 \times 1000}{2} = \text{USD } 20, \qquad n = 2 \times 1.5 = 3$$

Its payments at 0.5, 1.0, and 1.5 years are USD 20, USD 20, and USD 1,020.

Quarterly coupon amount

For a face value of USD 500, an annual coupon rate of 4.8%, and quarterly payments, the coupon is:

$$C = \frac{0.048 \times 500}{4} = \text{USD } 6$$

The maturity determines the number of payments. Under the level-coupon assumption, the maturity does not change the coupon amount.

2.1.6 Check your understanding

These items cover term interpretation, coupon arithmetic, and schedule construction. Each question stays collapsed until you open it; answers and explanations appear once you check.

Knowledge check 2.1.1: Fixed-rate bond contract and cash flows

1. A simplified bond has USD 1,000 face value, a 5% annual coupon rate, two coupon payments per year, and two years to maturity. Which field determines the principal redeemed at maturity under the lesson convention?

   (a) Face value
   (b) Annual coupon rate
   (c) Coupon frequency

2. A simplified fixed-rate bond's market price changes while its contract is unchanged. Which listed field remains a contractual input to promised coupon cash flows?

   (a) The annual coupon rate
   (b) The new market price
   (c) The size of the market-price change

3. Under the lesson convention, a bond has USD 1,000 face value, a 6% annual coupon rate, and two equal coupon payments per year. What is each coupon amount in USD?

4. Under the lesson convention, a bond has USD 2,000 face value, a 4.5% annual coupon rate, and two equal coupon payments per year. What is each coupon amount in USD?

5. From the holder's perspective, what are the promised cash flows of a two-year bond with USD 1,000 face value, a 5% annual coupon rate, and one coupon payment per year?

   (a) USD +50 after year one and USD +1,050 after year two
   (b) USD +50 after each year and no principal
   (c) USD +1,100 after year two only

6. From the holder's perspective, what are the promised cash flows of an 18-month bond with USD 1,000 face value, a 4% annual coupon rate, and semiannual coupons?

(a) USD +20, USD +20, and USD +1,020 at six-month intervals

(b) USD +40, USD +40, and USD +1,040 at six-month intervals

(c) USD +1,020 at every coupon date

Answers: *end of the chapter.*

2.1.7 Model boundary and review note

This lesson describes promised cash flows only. It does not assign a price, yield, payment probability, or expected recovery.

The contract terms and the level-coupon schedule follow Tuckman & Serrat[2]. The lesson stays `draft` pending human confirmation of the printed locators.

References

[1] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). Ch. 1 §1.1, printed p. 50 and Table 1.1.

[2] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022). §1.1, "Government Coupon Bonds": a bond is set by coupon rate, maturity date, and face (par, principal); each semiannual coupon is half the annual rate times face, with principal repaid at maturity (Table 1.1).

Notation

$\mathbf{1}_{\{k=n\}}$ **Final payment indicator**. Equals one on the final payment date, where the payment index equals the number of payments, and zero on every other date. It places the face value on the final date only.

c **Annual coupon rate**. Contractual annual rate used to determine a fixed-rate bond's coupon payments; applied to face value, not to market price. *Units:* decimal rate per year.

CF_k^{bond} **Bond cash flow**. Promised amount paid to the bondholder on one scheduled payment date; a positive receipt for the bondholder, with default excluded in the bond lessons. *Units:* stated currency at payment time.

m_{B} **Bond payment frequency**. Number of scheduled coupon payments per year in the simplified regular bond, a positive integer. *Units:* scheduled coupon payments per year.

k **Bond payment index**. Labels one remaining scheduled bond payment in increasing time order; it selects a payment and is not itself a time or currency amount. *Units:* dimensionless schedule index.

m **Compounding frequency**. Number of equal compounding periods per year under the stated rate convention, a positive integer fixed by the model convention. *Units:* compounding periods per year.

C **Coupon payment**. Level periodic cash amount promised by the simplified fixed-rate bond; a positive receipt for the bondholder in these lessons. *Units:* stated currency per coupon date.

F **Face value**. Contractual reference amount used to determine coupons and principal redemption. *Units:* stated currency.

T **Maturity time**. Final scheduled time when principal is redeemed in the simplified bond. *Units:* model-years from the valuation date.

n **Number of bond payments**. Counts the remaining regular coupon dates including maturity; a positive integer for the simplified regular bond schedule. *Units:* scheduled payment dates.

t_k **Payment time**. Time from the valuation date to one scheduled cash-flow event; a time coordinate, not a calendar date or payment index. *Units:* years from the valuation date.

0 **Valuation time**. Common origin from which later model times and present values are measured. *Units:* years from the valuation date.

Answers to the knowledge checks

Knowledge check 2.1.1: Fixed-rate bond contract and cash flows

1. **(a)** Face value. The simplified contract redeems principal at face value; the coupon rate and frequency determine periodic coupons.

2. **(a)** The annual coupon rate. In the simplified model, promised coupons follow the contractual face value, annual coupon rate, and frequency rather than the current market price.

3. 30 (accepted within $\pm 1e-9$). Coupon = face value $\times$ annual coupon rate / payments per year = 1,000 $\times$ 0.06 / 2 = 30.

4. 45 (accepted within $\pm 1e-9$). Coupon = 2,000 $\times$ 0.045 / 2 = 45.

5. **(a)** USD +50 after year one and USD +1,050 after year two. The coupon is 5% of face value each year; principal is added to the final coupon.

6. **(a)** USD +20, USD +20, and USD +1,020 at six-month intervals. Each semiannual coupon is 1,000 $\times$ 4% / 2 = 20, with principal paid only at maturity.