Chapter 2.6

Bond forwards

From *Credit Products Playground*, with changes after version 0.4.0, not yet released. Interactive version: https://danieltuzes.github.io/credit-and-derivatives/bonds/bond-forwards/

2.6.1 The forward contract

The holder of a long bond forward must pay the forward-delivery-price and receive the named bond at forward-delivery-time. The holder of the forward does not own the bond before delivery. So coupons paid before delivery belong to the current owner of the bond.

2.6.2 Cash-and-carry price

At valuation-time, start with the current dirty-bond-price P^{dirty}. Subtract the local predelivery-coupon-present-value I_0^{pre}. The difference is the prepaid forward value: the value today of receiving the bond at delivery. Dividing the prepaid forward value by the discount factor to delivery converts it to an amount at delivery. This amount is the fair bond-forward-price:

$$F^B_{0,T_{\text{fwd}}} = \frac{P^{\text{dirty}} - I_0^{\text{pre}}}{D(0, T_{\text{fwd}})} \tag{2.6.1}$$

For a dirty price of USD 960, a present value of the predelivery coupons of USD 95.45, and a discount factor to delivery of 0.92, (2.6.1) gives the bond forward price:

$$F^B_{0,T_{\text{fwd}}} = \frac{960 - 95.45}{0.92} = 939.72826087 \text{ USD per bond} \tag{2.6.2}$$

An existing contract can have a different delivery price. The earlier lesson on forward value calculates the value of such a contract. This lesson calculates the fair delivery price of a new contract.

2.6.3 Basis consistency

A clean price quote must first be converted to a dirty price. In the same way, the accrued interest at the delivery date must be added to a clean delivery price or a clean strike. If clean and dirty prices are mixed, the result can look like a pricing discrepancy, but it is only a mismatch of conventions.

Knowledge check 2.6.1: Bond forward

1. A bond's current dirty price is USD 960. The present value of coupons paid before delivery is USD 95.45, and D(0,T) = 0.92. Calculate the fair dirty bond forward delivery price in USD.

2. A bond's dirty spot price is USD 1,025, pre-delivery coupons have present value USD 24, and D(0,T) = 0.97. Calculate the fair dirty delivery price in USD.

Answers: end of the chapter.

2.6.4 Sources

The known-income cash-and-carry relation (2.6.1) and its coupon-bond application follow Hull, Chapter 5 §§5.3–5.5; Hull also treats the dirty-price basis of the bond forward and the strike in Chapter 28 §28.1 [1].

References

[1] Hull, Options, Futures, and Other Derivatives (8th ed., 2012).

Notation

I_0^{pre} **Predelivery coupon present value**. Valuation-time present value of coupons paid to the current bond owner before forward delivery. *Units:* stated currency at valuation time per bond.

$A(0,t)$ **Accumulation factor**. Grows one current unit over a stated future horizon under the selected rate model. *Units:* dimensionless currency-units per current currency-unit.

$F^B_{0,T_{\text{fwd}}}$ **Bond forward price**. Fair dirty delivery price fixed at valuation time for delivery of the named bond at the forward date; positive delivery cash paid by the long under the lesson's no-arbitrage assumptions. *Units:* stated currency at forward delivery per bond.

m **Compounding frequency**. Number of equal compounding periods per year under the stated rate convention, a positive integer fixed by the model convention. *Units:* compounding periods per year.

P^{dirty} **Dirty bond price**. Full cash or invoice price paid for the bond, equal to clean price plus accrued interest; a positive cash price paid by the buyer under the stated settlement convention. *Units:* stated currency at settlement.

$D(0,t)$ **Discount factor**. Converts one deterministic future unit into value at valuation time. *Units:* current currency-units per future currency-unit.

K_{fwd} **Forward delivery price**. Contractual currency amount per unit of underlying paid by the long at delivery and received by the short. *Units:* delivery-time currency per unit of underlying.

T_{fwd} **Forward delivery time**. Future model time when the forward counterparties exchange the underlying and delivery payment; a contract date shared by the long and short, not the underlying asset's maturity. *Units:* model-years from the stated valuation time.

$j^{(m)}$ **Nominal annual rate**. Annualized rate quote that must be paired with its compounding frequency. *Units:* decimal rate per year.

r_m **Periodic rate**. Rate applied once in each compounding period under the stated convention, derived from the stated nominal annual quote in this model. *Units:* decimal per compounding period.

0 **Valuation time**. Common origin from which later model times and present values are measured. *Units:* years from the valuation date.

Answers to the knowledge checks

Knowledge check 2.6.1: Bond forward

1. 939.7282608695652 (accepted within ± 0.000001). The prepaid forward is 960 - 95.45 = 864.55. Divide by 0.92 to obtain USD 939.72826087.

2. 1031.958762886598 (accepted within ± 0.000001). The fair delivery price is (1,025 - 24) / 0.97 = USD 1,031.95876289.