Chapter 5.2

Issuer-default knockout bond options

From *Credit Products Playground*, with changes after version 0.4.0, not yet released. Interactive version: https://danieltuzes.github.io/credit-and-derivatives/bond-options/issuer-default-knockout/

5.2.1 Contract definition

In this lesson, "knockout" has one trigger: if the issuer defaults before the exercise time, the option is extinguished and pays no rebate. This knockout is not a barrier on the market price of the bond.

The underlying bond is a different claim. The bond can still pay recovery after default. So the recovery is part of the bond value, and it is never part of the option value in the default state.

5.2.2 Value the underlying bond first

An alive node is a node at which the issuer has not defaulted. At an alive node with lattice-time-index i and lattice-state-index j, use the local node discount factor $d_{i,j}$, the local risk-neutral up weight $q_{i,j}$, the conditional-node-survival-probability $s_{i,j}$, the next surviving cash flow CF^B_{i+1}, and the local bond recovery $R^B_{i,j}$. The local surviving-bond-continuation-value is the bond value at the next time, conditional on survival. It is the next surviving cash flow plus the expected alive bond value:

$$H^B_{i,j} = CF^B_{i+1} + (1 - q_{i,j})B^{\text{alive}}_{i+1,j} + q_{i,j}B^{\text{alive}}_{i+1,j+1} \tag{5.2.1}$$

The Alive-state bond value is the discounted expectation over survival and default. Survival gives the continuation value from (5.2.1), and default gives the recovery:

$$B^{\text{alive}}_{i,j} = d_{i,j}\left[s_{i,j}H^B_{i,j} + \left(1 - s_{i,j}\right)R^B_{i,j}\right] \tag{5.2.2}$$

The values at each node are ex-cash-flow. The model assumes that recovery is paid at the next lattice date. This timing is a simplification.

5.2.3 Apply the knockout to option continuation

At call expiry, if the issuer has not defaulted, the option value is the call payoff:

$$O^{\text{KO}}_{T,j} = \max\left(B^{\text{alive}}_{T,j} - K, 0\right) \tag{5.2.3}$$

Before expiry, the option value in the default state is zero. So the option value at a node is the discounted, survival-weighted expectation of the option values at the two successor nodes:

$$O^{\mathrm{KO}}_{i,j} = d_{i,j}s_{i,j}\left[\left(1-q_{i,j}\right)O^{\mathrm{KO}}_{i+1,j} + q_{i,j}O^{\mathrm{KO}}_{i+1,j+1}\right] \tag{5.2.4}$$

(5.2.4) has no recovery term, unlike (5.2.2). If the conditional survival probability at the current node is zero, the option value is zero, also when the bond recovery is positive.

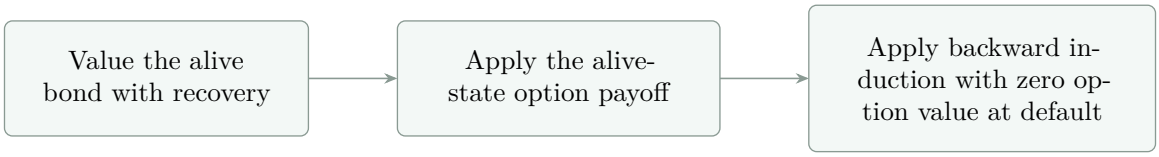


Diagram 5.2.1 **Issuer-default knockout valuation flow.** Value the underlying bond first, including its separate input recovery. At option expiry, apply the payoff only in alive states. Before expiry, discount only the survival-weighted option values, because the option value at default is zero.

5.2.4 Worked example

Consider a lattice with two annual steps. At option expiry, the input values of the final period give alive bond values of USD 82.80 and USD 88.65. With a call strike of USD 85, (5.2.3) gives terminal option values of USD 0 and USD 3.65. At valuation time, the discount factor is 0.95, the conditional survival probability is 0.90, and the up weight is 0.50. The option values are:

$$\begin{aligned} O^{\mathrm{KO}}_{1,0} &= \max(82.80-85,0) = 0 \\ O^{\mathrm{KO}}_{1,1} &= \max(88.65-85,0) = 3.65 \\ O^{\mathrm{KO}}_{0,0} &= 0.95(0.90)\left[(1-0.50)\,(0)+0.50\,(3.65)\right] \\ &= 1.560375 \text{ USD per option} \end{aligned} \tag{5.2.5}$$

The domain function `valueDefaultKnockoutBondOption` performs the same bond recursion and option recursion, in the same order.

5.2.5 Put-call parity with a default knockout

Put-call parity for options without a knockout does not hold unchanged for this contract. In a default state before expiry, the call value minus the put value is zero. So the matching synthetic forward must also be extinguished by the same default event. A default-free forward does not match the options in every state.

5.2.6 Model and source boundary

Hull describes European bond options. Separately, Hull describes CDS forwards and CDS options that cease to exist if the reference entity defaults. This lesson combines the two descriptions for teaching: an option on the issuer's bond that is knocked out by issuer default. The lesson implements the stated contract. It does not claim that all bond options use this convention. The joint lattice for rates and credit is an input; it is not calibrated.

Knowledge check 5.2.1: Issuer-default knockout bond option

1. In this lesson's contract, what happens if the issuer defaults before option exercise?
   (a) The option is extinguished with zero option rebate
   (b) The option holder automatically receives the bond's recovery amount
   (c) The option survives unless the bond price also crosses a barrier
2. The bond pays USD 40 recovery after default, but an otherwise identical call is extinguished on that default. What enters the option's default branch?
   (a) USD 0; the USD 40 belongs only in the bond valuation
   (b) USD 40 because every bond cash flow passes through to the option
   (c) The option strike because exercise is forced
3. At an alive node, the risk-neutral expected next option value conditional on survival is USD 10, conditional survival over the period is 0.90, and the node discount factor is 0.95. Default gives zero option rebate. Calculate the current option value in USD.
4. At an alive node, next option values conditional on survival are USD 2 down and USD 8 up. The risk-neutral up weight is 0.25, conditional survival is 0.80, and the discount factor is 0.96. Calculate the current knockout option value in USD.

Answers: *end of the chapter.*

5.2.7 Sources

The bond-option cash-price conventions follow Hull, Chapter 28 §28.1. Hull describes the analogous extinguishment at default for CDS forwards and CDS options in Chapter 24 §24.5. Backward induction follows Chapter 12 [1]. A recovery convention must state its base and its timing, as the fixed-income treatment emphasizes [2].

References

[1] Hull, Options, Futures, and Other Derivatives (8th ed., 2012).

[2] Tuckman & Serrat, Fixed Income Securities: Tools for Today's Markets (4th ed., 2022).

Notation

$R^B_{i,j}$ **Bond default recovery cash**. Non-negative recovery paid on the underlying bond at the next lattice date after default in the period. *Units:* stated currency per bond at the next time.

$H^B_{i,j}$ **Surviving bond continuation value**. Next-time scheduled bond cash plus risk-neutral expected ex-cash-flow bond value, conditional on period survival. *Units:* stated currency per bond at the next time.

CF^{B}_{i+1} **Surviving bond next cash flow**. Coupon or coupon-plus-redemption paid at the next lattice time only if the issuer survives the period. *Units:* stated currency per bond at the next time.

$B^{\text{alive}}_{i,j}$ **Alive-state bond value**. Ex-cash-flow value of the defaultable bond at a lattice node conditional on the issuer still being alive, before the subsequent survival and default branches; scheduled cash at the node has already been paid. *Units:* stated currency per bond at the node time.

$s_{i,j}$ **Conditional node survival probability**. Pricing-model probability that the issuer survives the next lattice period, conditional on being alive at the current node; a risk-neutral input, not an unconditional real-world forecast. *Units:* probability between zero and one.

$O^{\text{KO}}_{i,j}$ **Knockout bond option value**. Alive-node value of a European bond option that is extinguished with zero option rebate by issuer default before exercise; a non-negative holder value conditional on the issuer being alive at the node. *Units:* stated currency per option at the node time.

$d_{i,j}$ **Lattice node discount factor**. One-period discount factor at a lattice node: currency at the node per one unit of currency at either successor node one step later. *Units:* current-node currency per next-time currency.

$q_{i,j}$ **Lattice node up weight**. Pricing probability assigned to the up successor, conditional on the current node; in a lattice with issuer default, also conditional on survival to the next lattice time. *Units:* probability between zero and one.

j **Lattice state index**. Integer label for one state node within a time row of a finite recombining lattice; a bookkeeping label under the stated successor ordering, not a probability or state value. *Units:* dimensionless integer index.

i **Lattice time index**. Integer label for one time row in a finite recombining valuation lattice; a bookkeeping label, not a model-year time or currency amount. *Units:* dimensionless integer index.

K **Option strike price**. Contractual price per unit of underlying used to determine the option's exercise payoff; a positive contractual amount, with payoff signs depending on call or put and holder or writer perspective. *Units:* expiry-time currency per unit of underlying.

Answers to the knowledge checks

Knowledge check 5.2.1: Issuer-default knockout bond option

1. **(a)** The option is extinguished with zero option rebate. Issuer default is the knockout event. Recovery on the underlying bond is modeled separately and is not an option rebate.

2. **(a)** USD 0; the USD 40 belongs only in the bond valuation. The contract separates underlying recovery from option termination.

3. 8.55 (accepted within ± 0.000001). The value is $0.95 \times [0.90 \times 10 + 0.10 \times 0] =$ USD 8.55.

4. 2.688 (accepted within ± 0.000001). The survival-state expectation is $0.75 \times 2 + 0.25 \times 8 = 3.5$. Multiply by survival and discounting: $0.96 \times 0.80 \times 3.5 =$ USD 2.688.