Chapter 5.1

European bond options

From *Credit Products Playground*, with changes after version 0.4.0, not yet released. Interactive version: https://danieltuzes.github.io/credit-and-derivatives/bond-options/european-bond-options/

5.1.1 The standalone contract

A European bond call gives the holder the right, but not the obligation, to buy a named bond at option-expiry-time. A put gives the right to sell. This option is not the issuer's call feature inside a callable bond. The two rights are held by different parties and belong to different instruments.

5.1.2 Put the bond value and the strike on one price basis

The local bond-option-expiry-value $B^{\text{dirty}}_{T_{\text{opt}}}$ and option-strike-price K must both be dirty cash prices. If a contract states a clean strike, add the accrued-interest at the exercise date to the strike before the comparison.

In this lesson, exercise occurs immediately after any coupon paid on the expiry date. The call payoff and the put payoff are:

$$X^{B,\text{call}}_{T_{\text{opt}}} = \max\left(B^{\text{dirty}}_{T_{\text{opt}}} - K, 0\right) \tag{5.1.1}$$

$$X^{B,\text{put}}_{T_{\text{opt}}} = \max\left(K - B^{\text{dirty}}_{T_{\text{opt}}}, 0\right) \tag{5.1.2}$$

The lattice first gives the bond values at expiry. Apply (5.1.1) or (5.1.2) at the expiry nodes. Then apply backward induction to calculate the lattice-node-value at time zero.

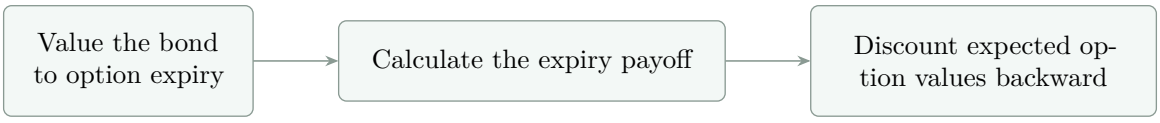


Diagram 5.1.1 **European bond-option valuation flow.** First value the bond at every option-expiry node, then apply the call or put payoff, then discount the risk-neutral expected payoffs backward through the input lattice.

5.1.3 Worked example

Suppose a one-period input lattice gives dirty bond values at expiry of USD 90 and USD 110. The strike is USD 100, the risk-neutral up weight is 0.60, and the discount factor is 0.95. Applying (5.1.1) and one backward step gives the call value:

$$\begin{aligned} X_{T_{\text{opt}}}^{B,\text{call,down}} &= \max(90-100,0) = 0 \\ X_{T_{\text{opt}}}^{B,\text{call,up}} &= \max(110-100,0) = 10 \\ V_0 &= 0.95\left[(1-0.60)\,(0)+0.60\,(10)\right] \\ &= 5.70 \text{ USD per option} \end{aligned} \tag{5.1.3}$$

The tested domain code does the same two steps: `europeanOptionPayoff` calculates the expiry payoffs, and `backwardInductionValue` discounts them.

5.1.4 Coupon and exercise ordering

In this lesson, a coupon on the expiry date is paid before exercise. So the bond value at expiry is ex-coupon. A contract that is exercised before the coupon, or that transfers the right to the coupon, requires different terminal inputs.

Knowledge check 5.1.1: European bond option

1. A standalone European call on a bond gives its holder which right?
   (a) The right, but not obligation, to buy the named bond at the stated strike and exercise time
   (b) The issuer's right to redeem every outstanding bond
   (c) Automatic ownership of every coupon paid before option exercise
2. An option strike is quoted as a clean bond price and exercise occurs between coupon dates. What is needed before comparing it with a dirty bond value?
   (a) Add the exercise-date accrued interest under the stated convention
   (b) Subtract every coupon paid over the bond's entire life
   (c) No adjustment; clean and dirty prices are identical between coupon dates
3. At expiry a bond has dirty ex-coupon values of USD 78, USD 84, and USD 89 across three nodes. Calculate call payoffs for strike USD 82 and report their sum in USD.
4. One period before expiry, a default-free bond call is worth USD 0 in the down node and USD 6 in the up node. The risk-neutral up weight is 0.4 and the node discount factor is 0.95. Calculate the current option value in USD.

Answers: end of the chapter.

5.1.5 Sources

The bond-option definition and the need to distinguish a cash (dirty) strike from a quoted (clean) strike follow Hull, Chapter 28 §28.1 [1]. This lesson uses an input lattice, not Black's market model, so that the next lesson can model the issuer-default state explicitly.

References

[1] Hull, Options, Futures, and Other Derivatives (8th ed., 2012).

Notation

$X^{B,\text{call}}_{T_{\text{opt}}}$ **Bond call expiry payoff**. Non-negative holder payoff from buying the bond at the dirty strike when economically beneficial. *Units:* stated currency per option at expiry.

$B^{\text{dirty}}_{T_{\text{opt}}}$ **Bond option expiry value**. Ex-coupon dirty cash value of the named bond at the European exercise instant. *Units:* stated currency per bond at option expiry.

$X^{B,\text{put}}_{T_{\text{opt}}}$ **Bond put expiry payoff**. Non-negative holder payoff from selling the bond at the dirty strike when economically beneficial. *Units:* stated currency per option at expiry.

AI **Accrued interest**. Coupon amount attributed to the interval from the previous coupon date through settlement under the stated day-count convention; a positive amount added to the clean price to obtain the dirty invoice price. *Units:* stated currency at settlement.

$V_{i,j}$ **Lattice node value**. Claim value at time row i and state node j obtained by one-period backward induction from its successor nodes, conditional on reaching that node under the input pricing lattice. *Units:* stated currency at the node time.

T_{opt} **Option expiry time**. Future model time when a European option's exercise decision and payoff are determined; a contract date shared by holder and writer, distinct from a bond maturity. *Units:* model-years from the stated valuation time.

K **Option strike price**. Contractual price per unit of underlying used to determine the option's exercise payoff; a positive contractual amount, with payoff signs depending on call or put and holder or writer perspective. *Units:* expiry-time currency per unit of underlying.

Answers to the knowledge checks

Knowledge check 5.1.1: European bond option

1. **(a)** The right, but not obligation, to buy the named bond at the stated strike and exercise time. This lesson models a standalone holder option, not an embedded issuer call feature.

2. **(a)** Add the exercise-date accrued interest under the stated convention. Payoff inputs must share one price basis. A clean strike becomes a dirty cash strike after adding applicable accrued interest.

3. 9 (accepted within ± 0.000001). The node payoffs are 0, 2, and 7, whose sum is USD 9. The sum is requested only as a compact payoff check, not as a probability-weighted value.

4. 2.28 (accepted within ± 0.000001). Backward induction gives $0.95 \times (0.6 \times 0 + 0.4 \times 6) =$ USD 2.28.